Spin physics: transverse theory and overview

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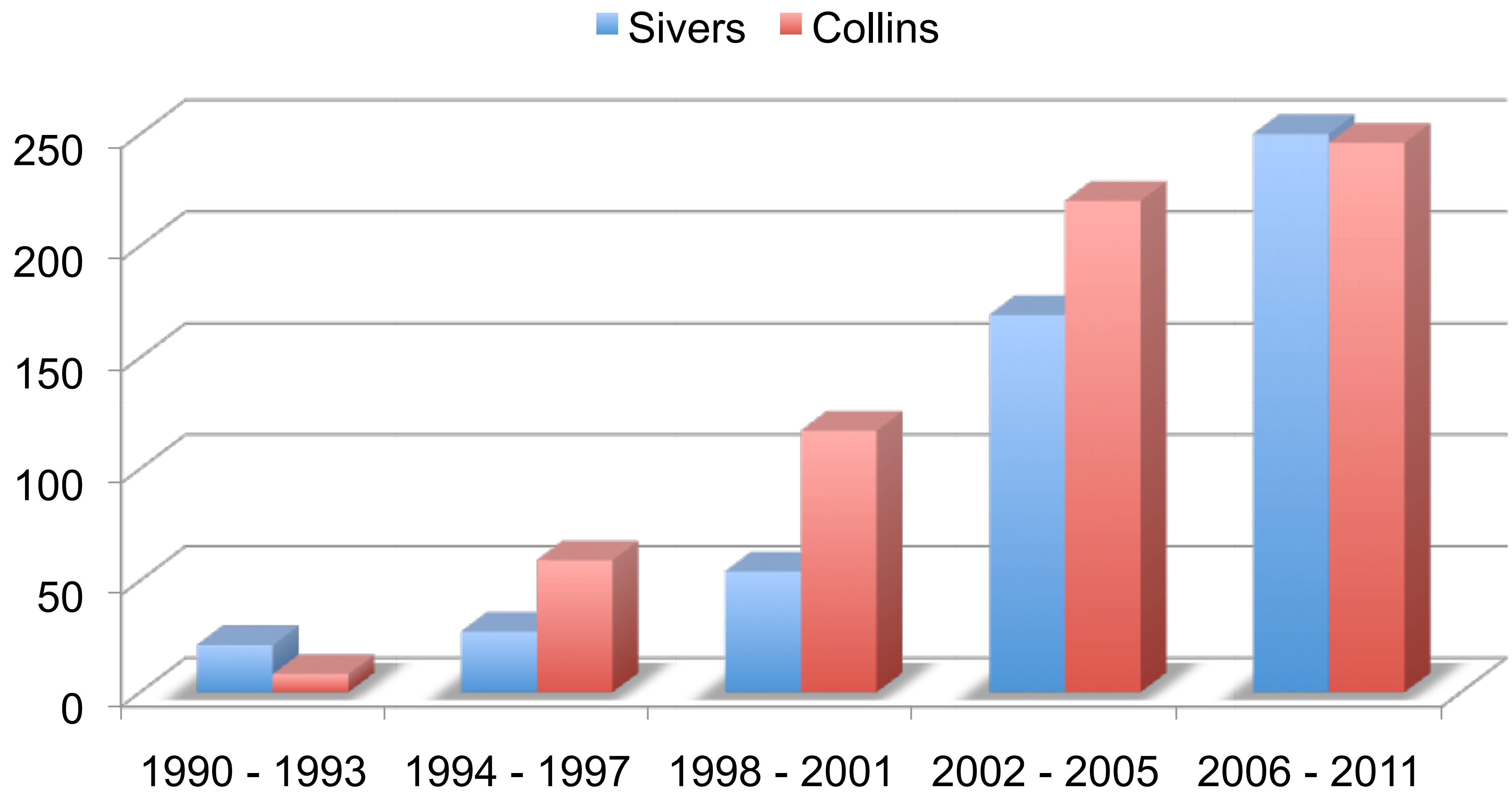
Outline

- Single transverse spin asymmetry: Sivers effect
- Process dependence of the Sivers function
- Global fitting of SIDIS and pp data
- QCD evolution and resummation
- Connection to small-x physics?
- Summary

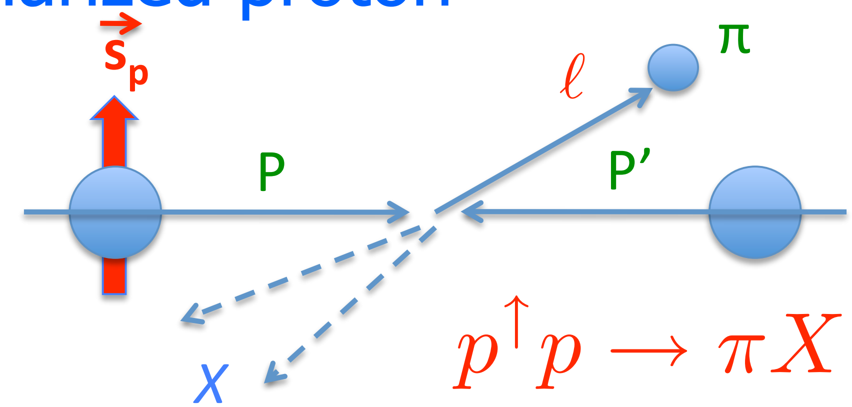
Transverse spin physics: birth and growth

- Historically transverse spin physics has been a source of much controversy
  - Early days (before 1980s), it was thought to be not very interesting
  - Recently it has become a very active research branch
- Citations tell the story: Sivers and Collins function - birth and growth



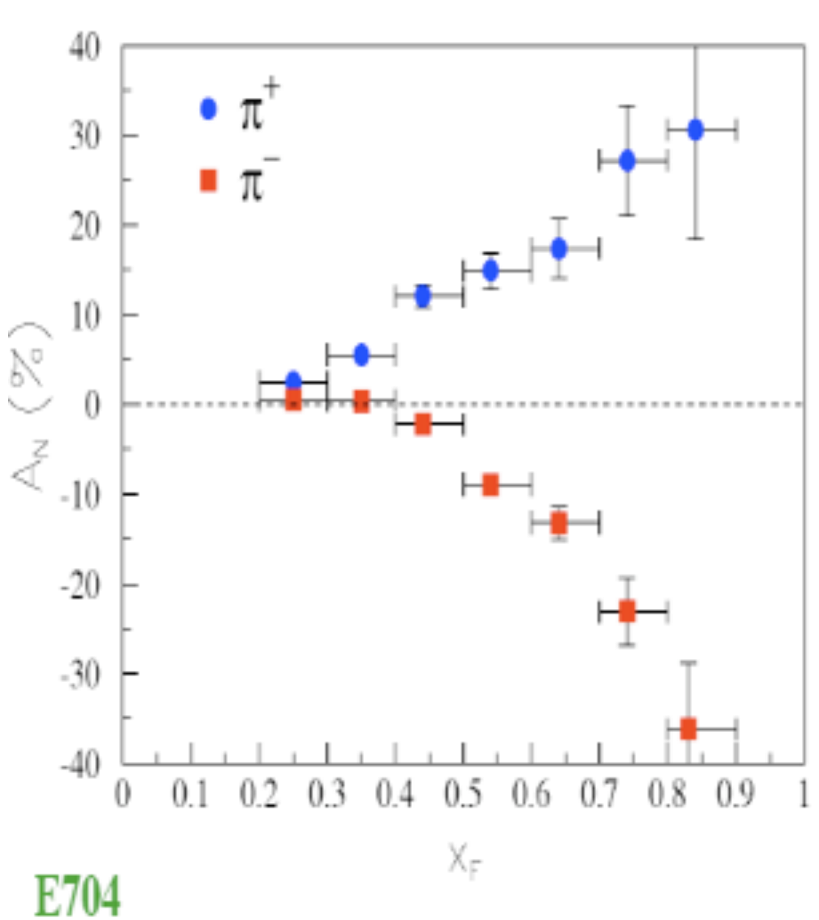
Single transverse spin asymmetry (SSA)

- Consider a transversely polarized proton scatters with another unpolarized proton



$$p^{\uparrow}p \rightarrow \pi X$$

$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$



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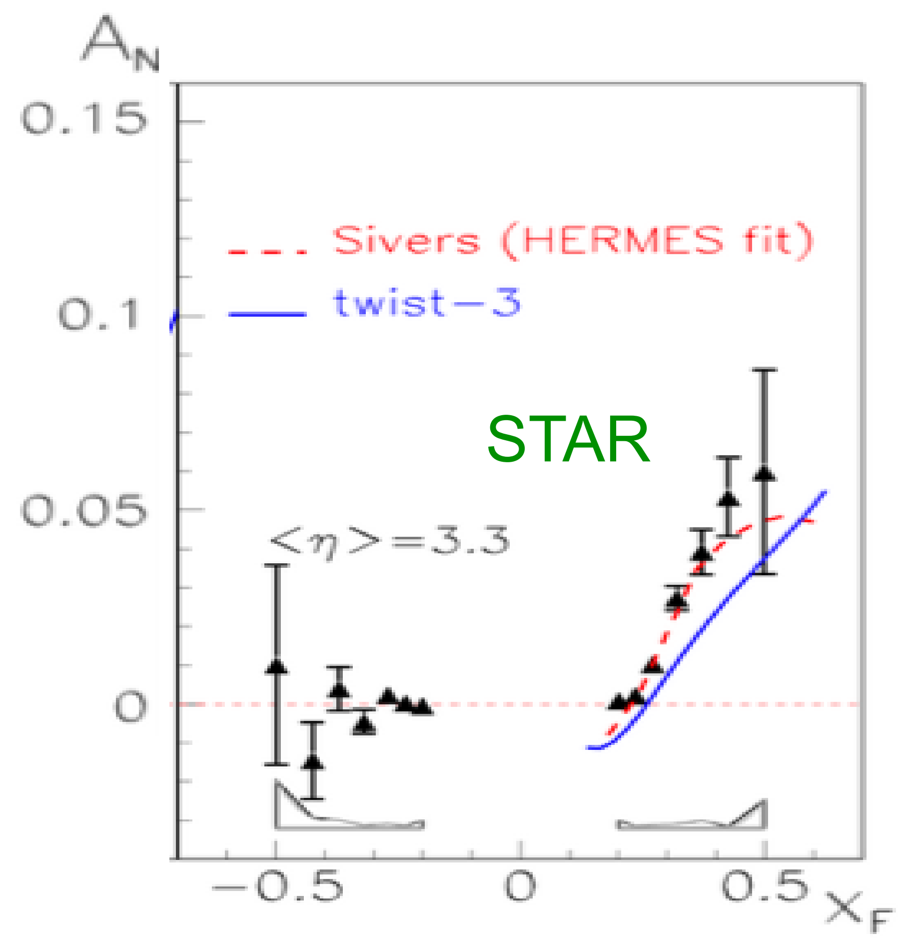
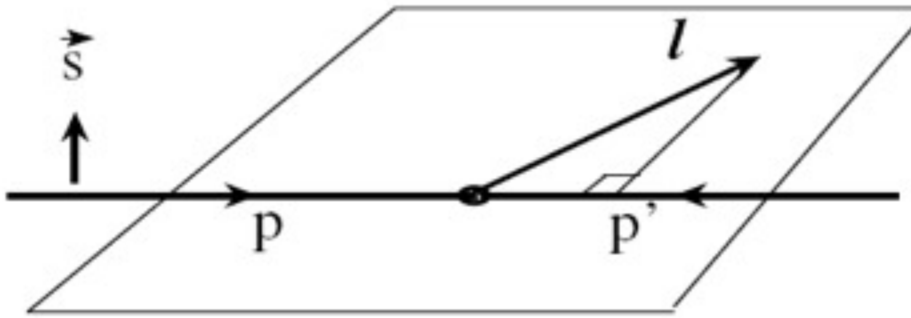


PHENIX
HERMES
COMPASS
JLAB, too

SSA corresponds to a T-odd triplet product

- SSA measures the correlation between the hadron spin and the production plane, which corresponds to $\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$

$$p^{\uparrow} p \to \pi(\ell) X$$



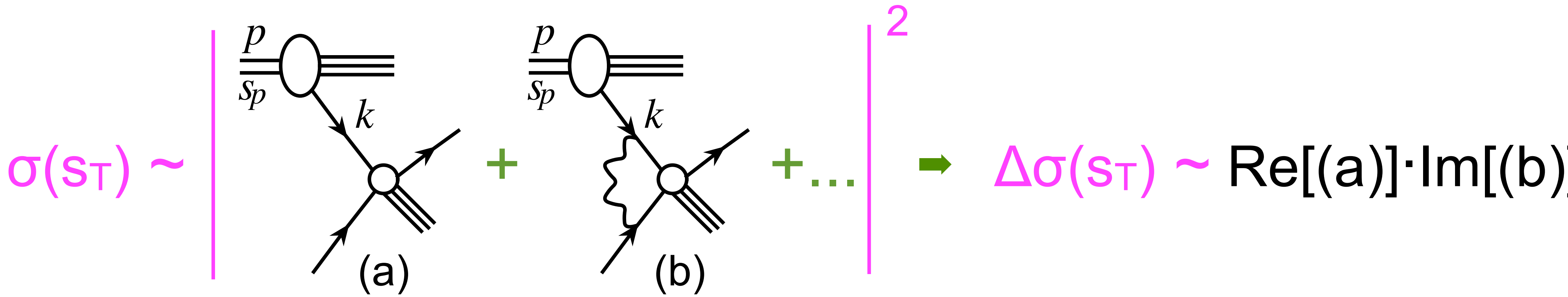
- Such a product is (naive) odd under time reversal (T-odd), thus they can arise in a time-reversal invariant theory (eg, QCD) only when there is a phase between different spin amplitudes

Nonvanishing A_N requires
a phase
a helicity flip
enough vectors to fix a scattering plane

SSA vanishes at leading twist in collinear factorization

- At leading twist formalism: partons are collinear

Kane, Pumplin, Repko, 1978



  - generate phase from loop diagrams, proportional to α_s
  - helicity is conserved for massless partons, helicity-flip is proportional to current quark mass m_q

Therefore we have

$$A_N \sim \alpha_s \frac{m_q}{P_T} \to 0$$

- $A_N \neq 0$: result of parton's transverse motion or correlations!

Two mechanisms to generate SSA in QCD

- TMD approach: Transverse Momentum Dependent distributions probe the parton's intrinsic transverse momentum

$$\sigma(p_h, s_\perp) \propto f_{a/A}(x, k_\perp) \otimes D_{h/c}(z, p_\perp) \otimes \hat{\sigma}_{\rm parton}$$

  - Sivers function: in Parton Distribution Function (PDF)

    Sivers 90

  - Collins function: in Fragmentation Function (FF)

    Collins 93

- Collinear twist-3 factorization approach: net K_T information

$$\sigma(p_h, s_\perp) \propto \frac{1}{Q} f_{a/A}^{s_\perp}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\rm parton}$$

  - Twist-3 three-parton correlation functions: Qiu-Sterman matrix element, ...

    Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98, ...

  - Twist-3 three-parton fragmentation functions:

    Koike, 02, Kang-Yuan-Zhou 2010, ...

Relation between twist-3 and TMD approaches

- They apply in different kinematic domain:
  - TMD approach: need TMD factorization, applies for the process with two observed momentum scales: DY at small $Q_T << Q$

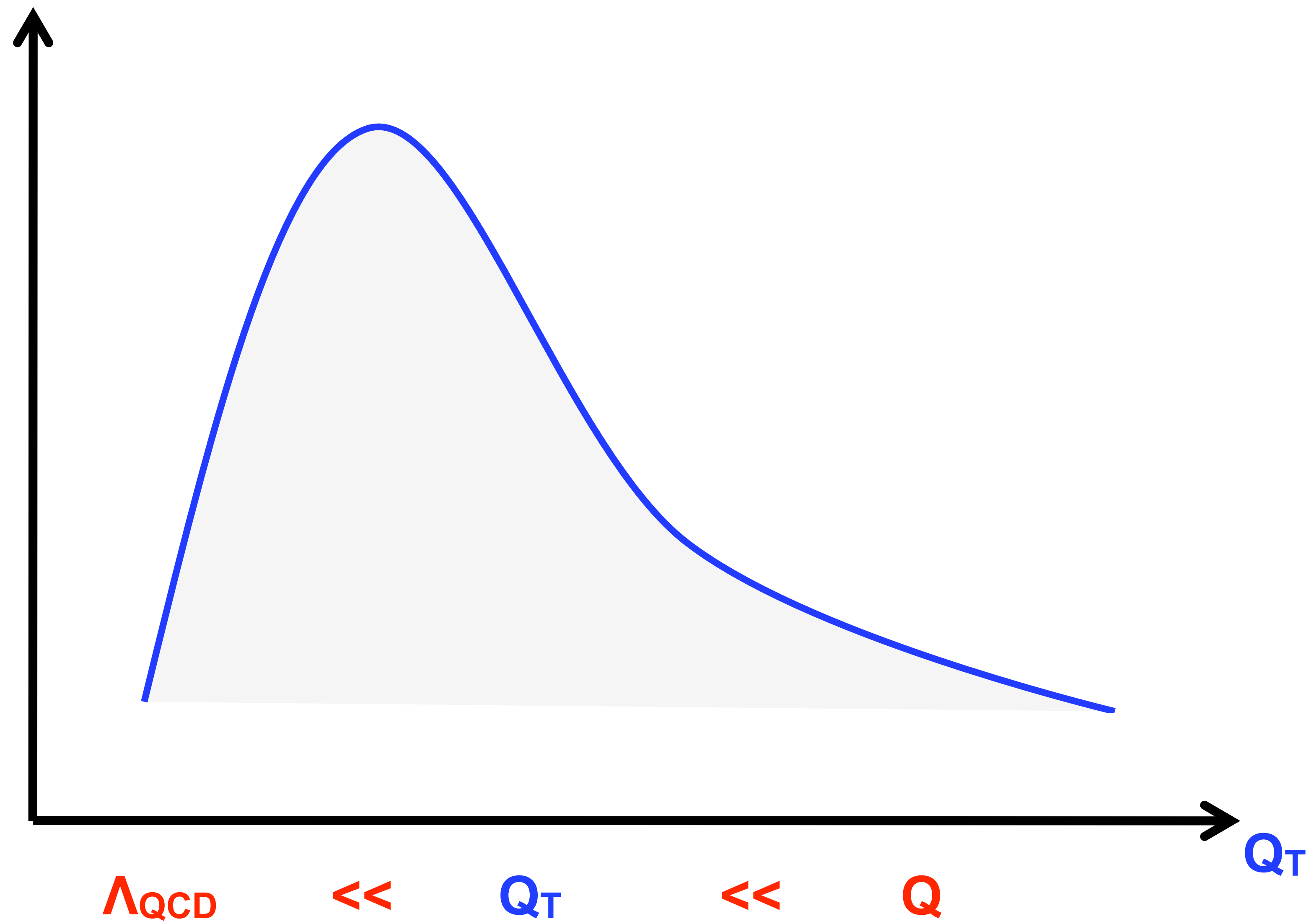
$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse momentum} \end{cases}$$

  - Collinear factorization approach: more relevant for single scale hard process inclusive pion production at high p_T in pp collision
- They generate same results in the overlap region when they both apply:
  - Twist-3 three-parton correlation in distribution $\longleftrightarrow$ Sivers function

    Ji-Qiu-Vogelsang-Yuan, 2006, …
  - Twist-3 three-parton correlation in fragmentation $\longleftrightarrow$ Collins function

    Koike 2002, Zhou-Yuan, 2009, Kang-Yuan-Zhou, 2010, …

A unified picture for Drell-Yan (leading Q_T/Q)



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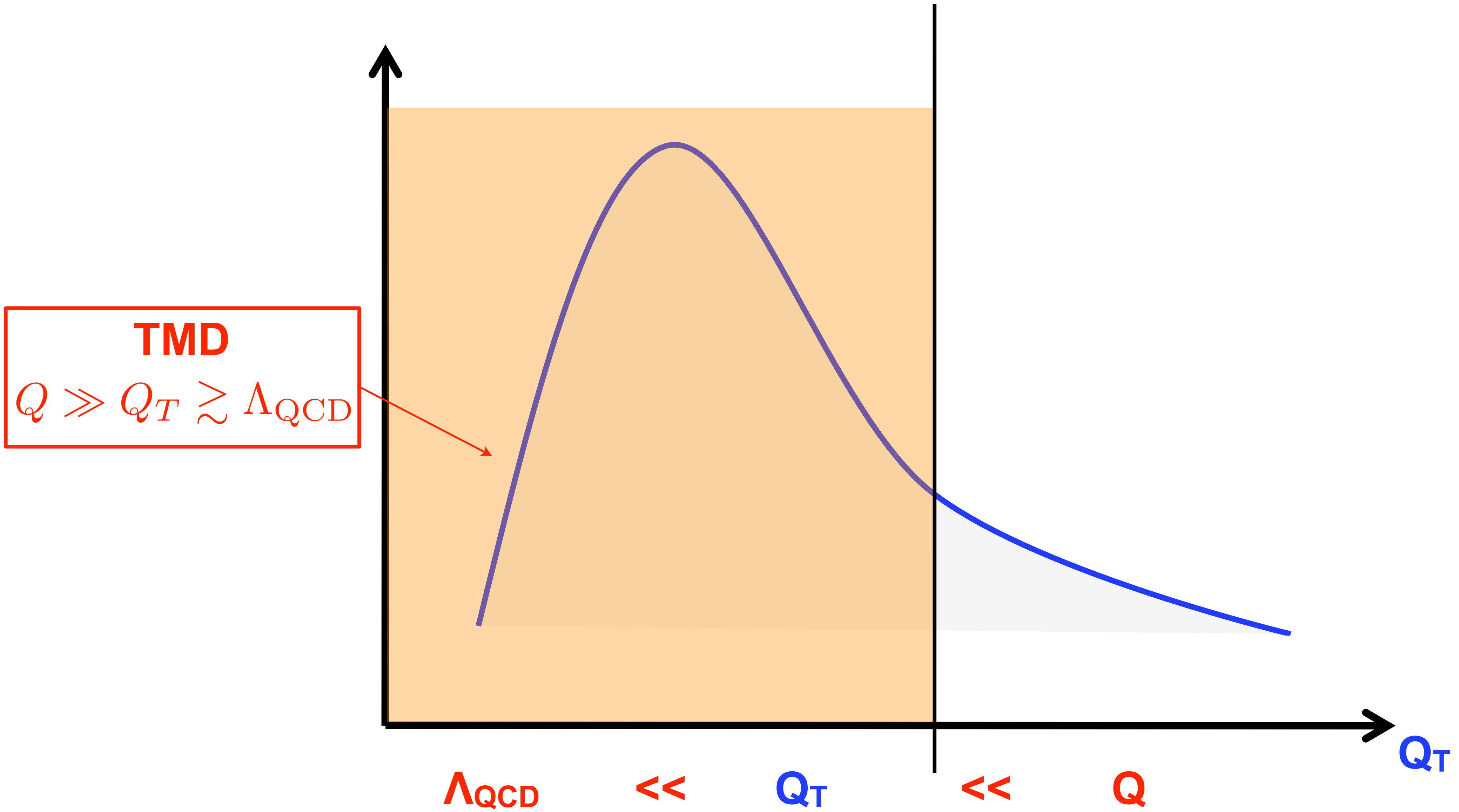

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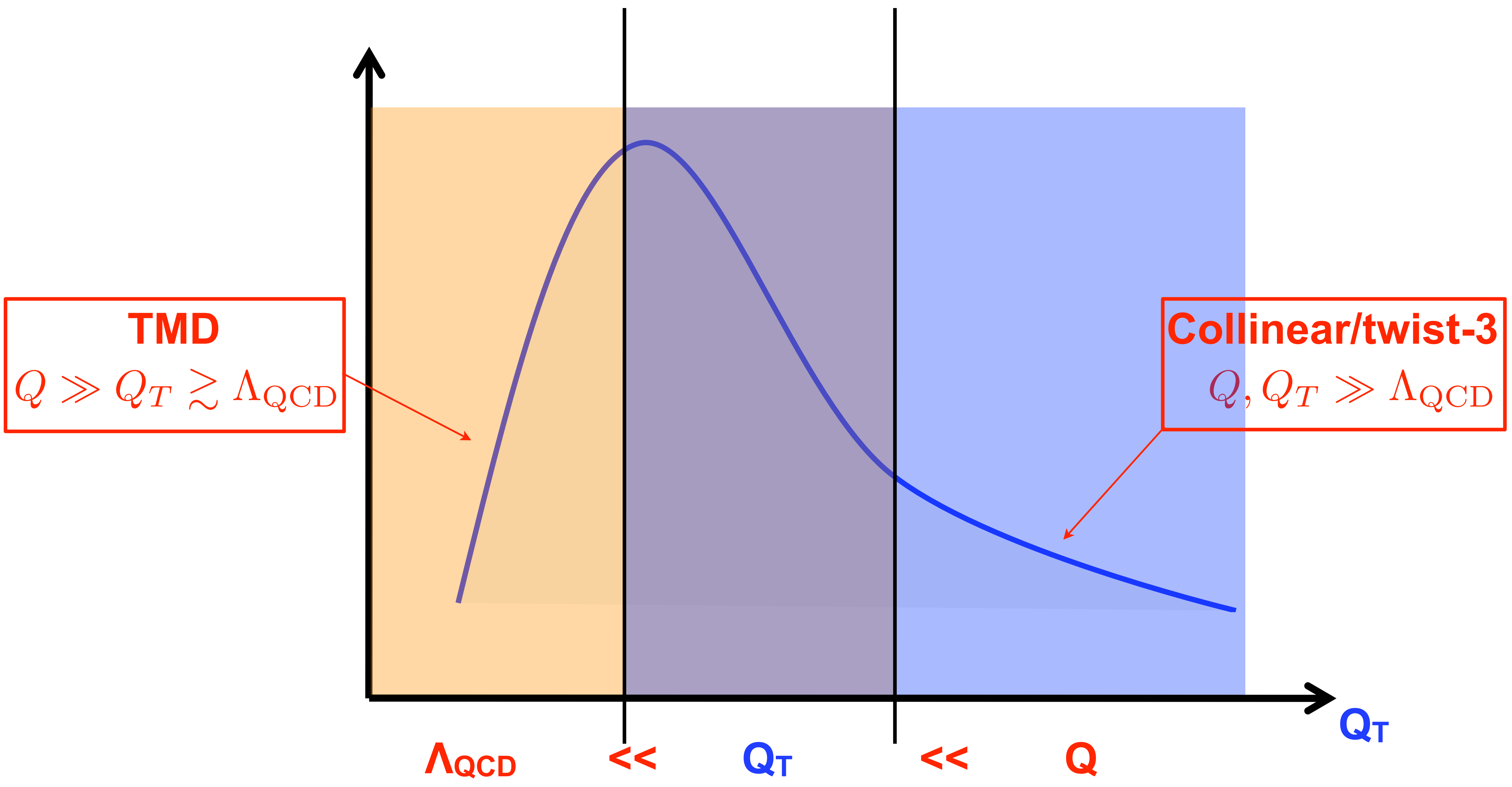
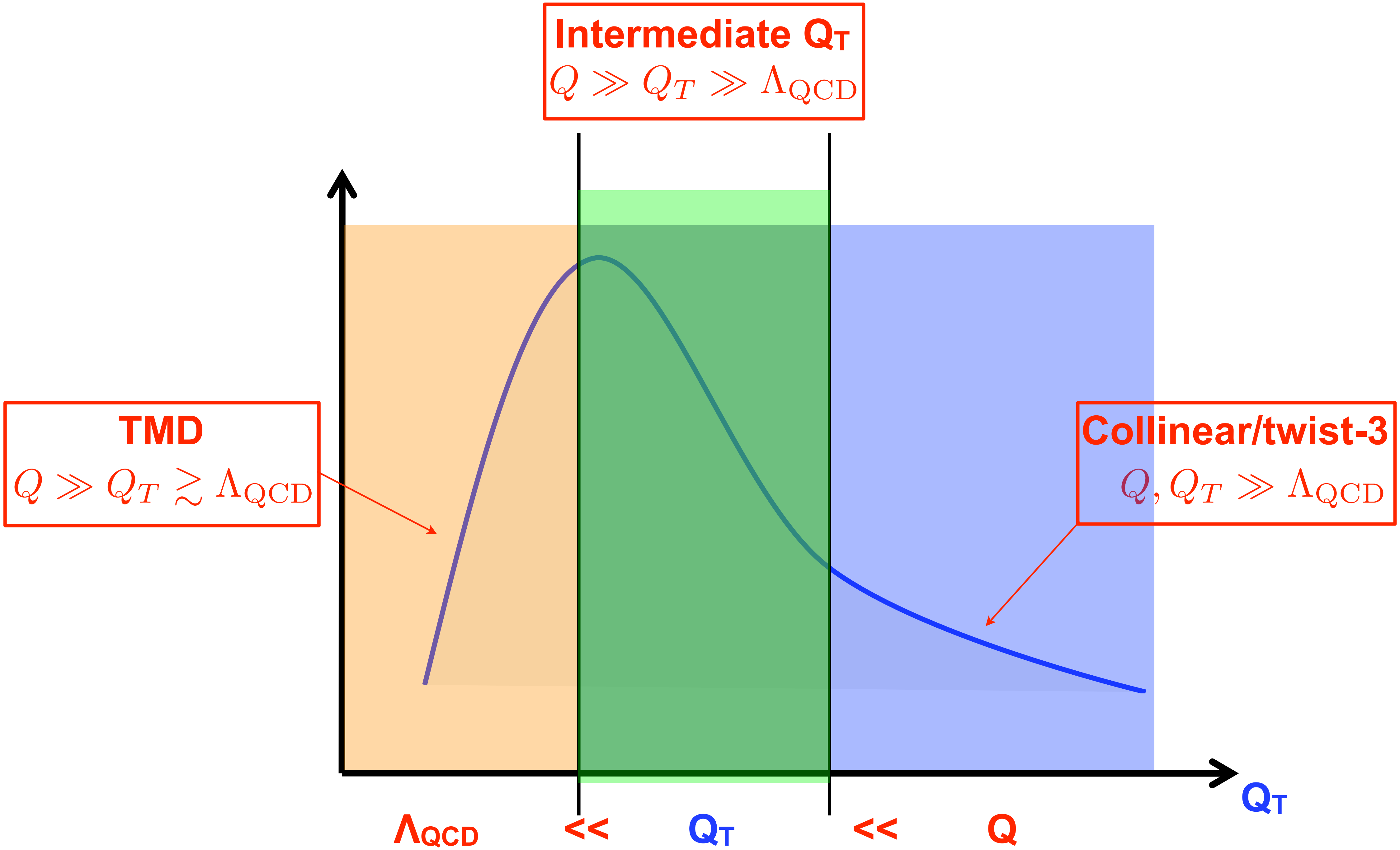


A unified picture for Drell-Yan (leading Q_T/Q)



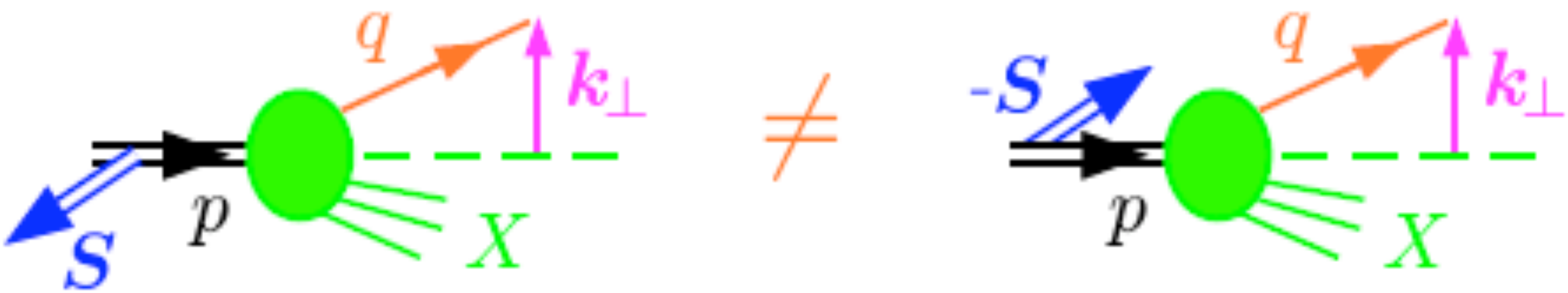
Transverse momentum dependent distribution (TMD)

- Sivers function: an asymmetric parton distribution in a polarized hadron (kt correlated with the spin of the hadron)

Spin-dependent

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

Spin-independent



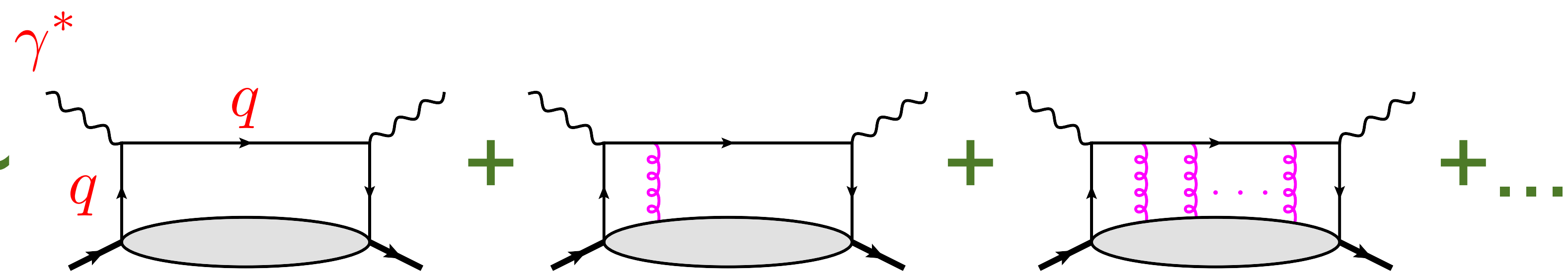
- Where does the phase come from?

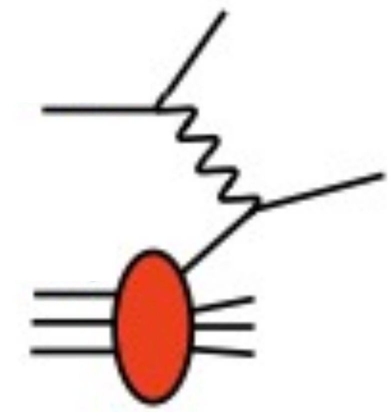
The history of Sivers function

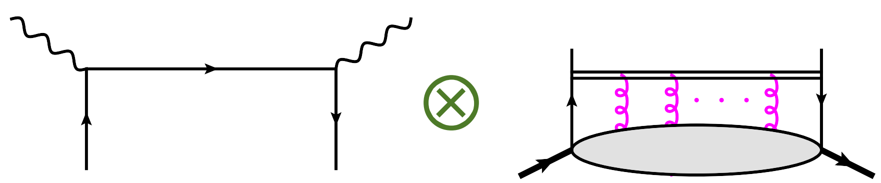
- 1990: Sivers function
  - introduce kt dependence of PDFs, generate the SSA through a correlation between the hadron spin and the parton kt
- 1993: Collins
  - show Sivers function vanishes due to time-reversal invariance
- 2002: Brodsky, Hwang, Schmidt
  - explicit model calculation show the existence of the Sivers function
  - the existence of Sivers function relies on the initial- and final-state interactions between the active parton and the remnant of the polarized hadron
- 2002: Ji, Yuan, Belitsky
  - the initial- and final-state interaction presented by Brodsky, et.al. is equivalent to the color gauge links in the definition of the TMD distribution functions
  - since the details of the initial- and final-state interaction depend on the specific scattering process, the gauge link thus the Sivers function could be process-dependent

Sivers function are process-dependent

- Existence of the Sivers function relies on the interaction between the active parton and the remnant of the hadron (process-dependent)
  - SIDIS: final-state interaction

$\sigma \sim$ 

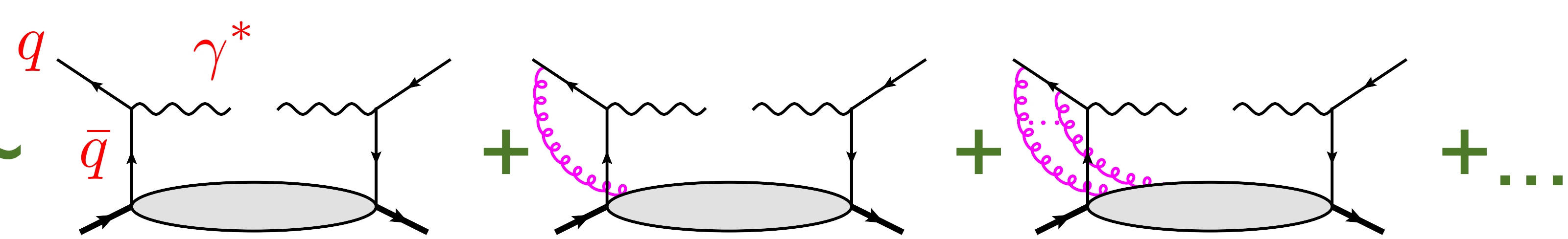


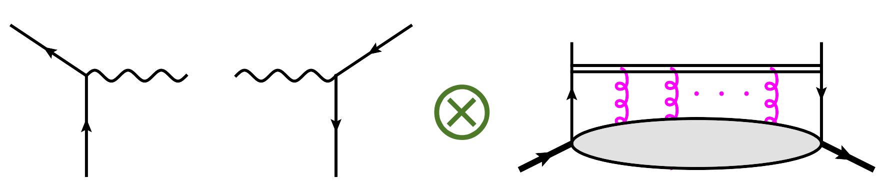
$\approx$ 

PDFs with SIDIS gauge link

$$\mathcal{P}\, e^{ig\int_y^{\infty} d\lambda\cdot A(\lambda)}$$

  - Drell-Yan: initial-state interaction

$\sigma \sim$ 

$\approx$ 

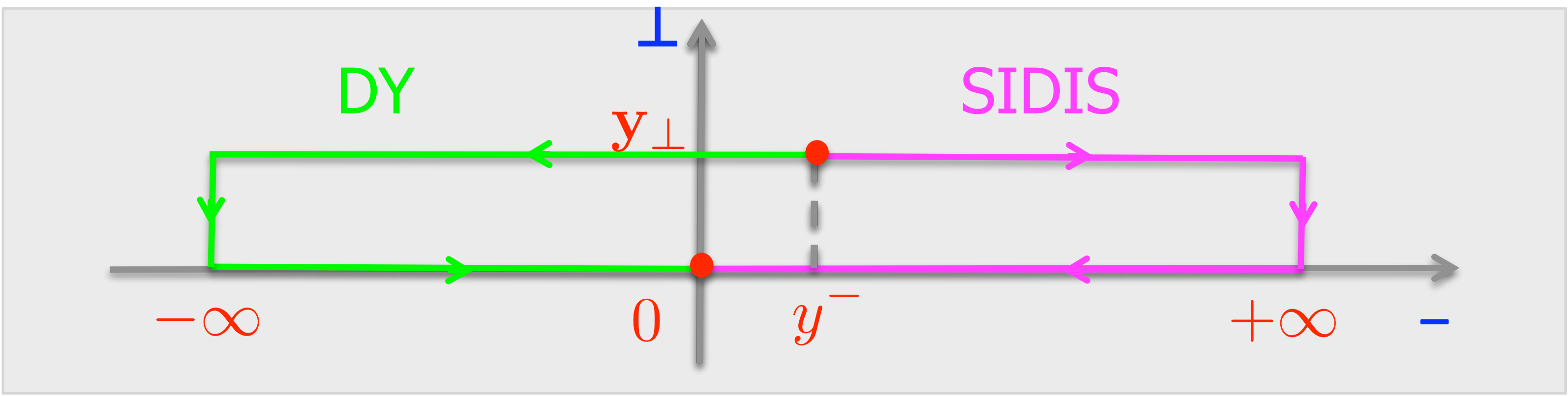
PDFs with DY gauge link

$$\mathcal{P}\, e^{ig\int_y^{-\infty} d\lambda\cdot A(\lambda)}$$

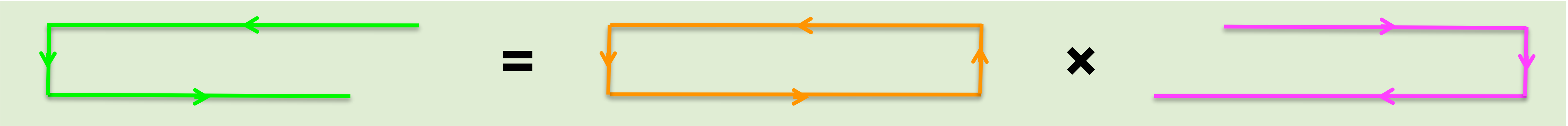
Time-reversal modified universality of the Sivers function

- Different gauge link for gauge-invariant TMD distribution in SIDIS and DY

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$



Wilson Loop ~ $\exp\left[-ig\int_\Sigma d\sigma^{\mu\nu}F_{\mu\nu}\right]$ Area is NOT zero



- Parity and time-reversal invariance:

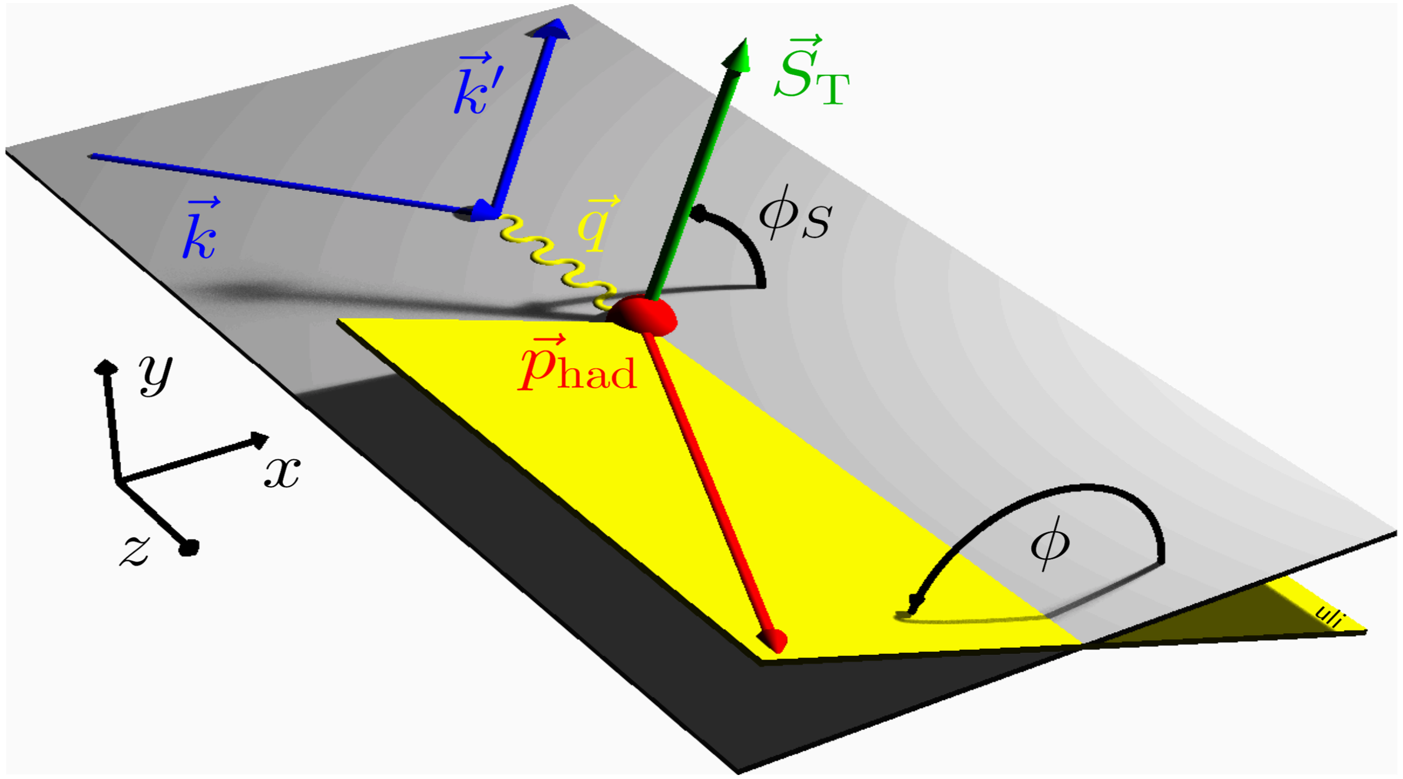
$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\text{DY}}(x,k_\perp)$$

Most critical test for TMD approach to SSA

Current Sivers function from SIDIS

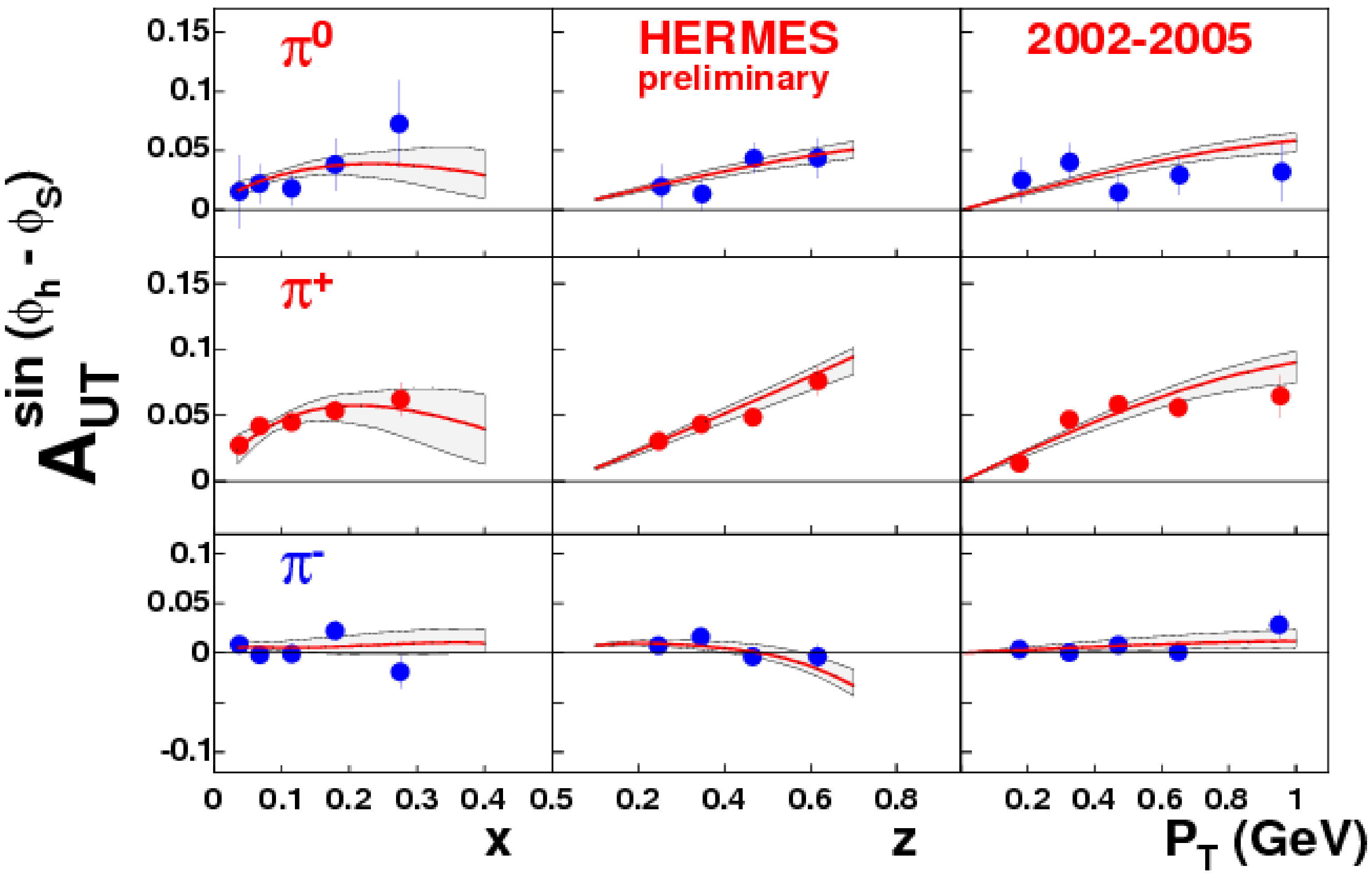
- Sivers and Collins can be separately extracted from SIDIS

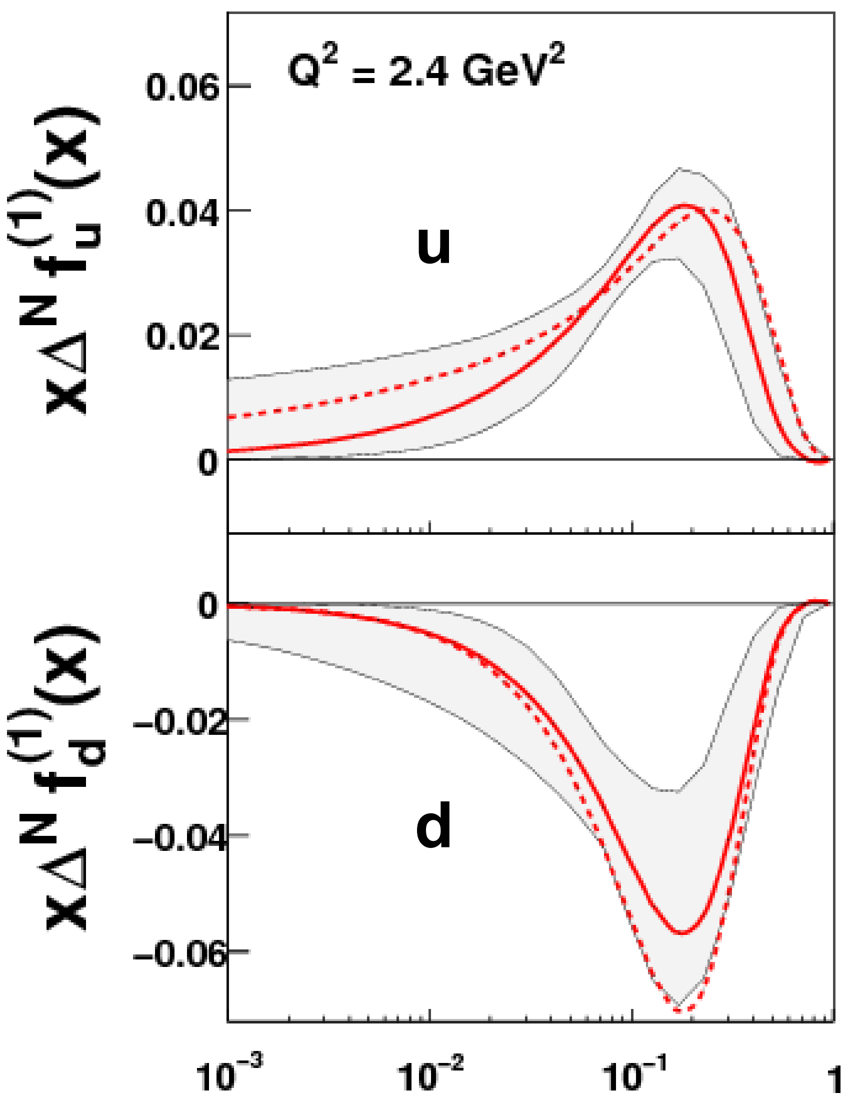
$$\Delta\sigma \propto A_{UT}^{Collins}\sin(\phi+\phi_S) + A_{UT}^{Sivers}\sin(\phi-\phi_S)$$



Sivers function from SIDIS $\ell + p^\uparrow \to \ell' + \pi(p_T) + X: \quad p_T \ll Q$

- Extract Sivers function from SIDIS (HERMES&COMPASS)





Anselmino, et.al., 2009

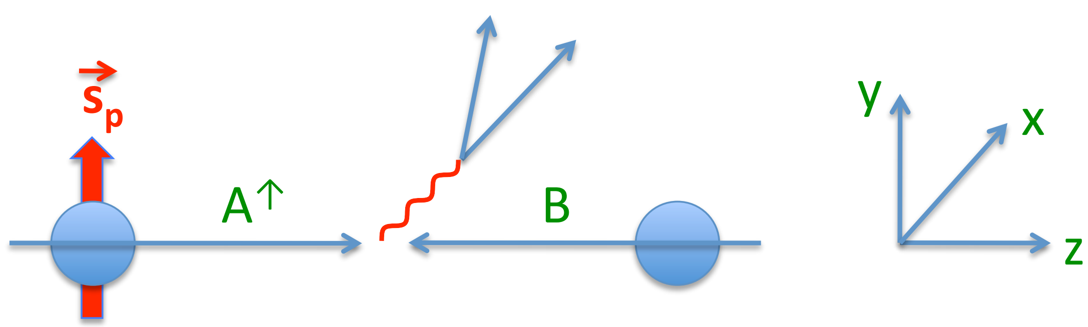
  - u and d almost equal size, different sign
  - d-Sivers is slightly larger

- Still needs DY results to verify the sign change, thus fully understand the mechanism of the SSAs

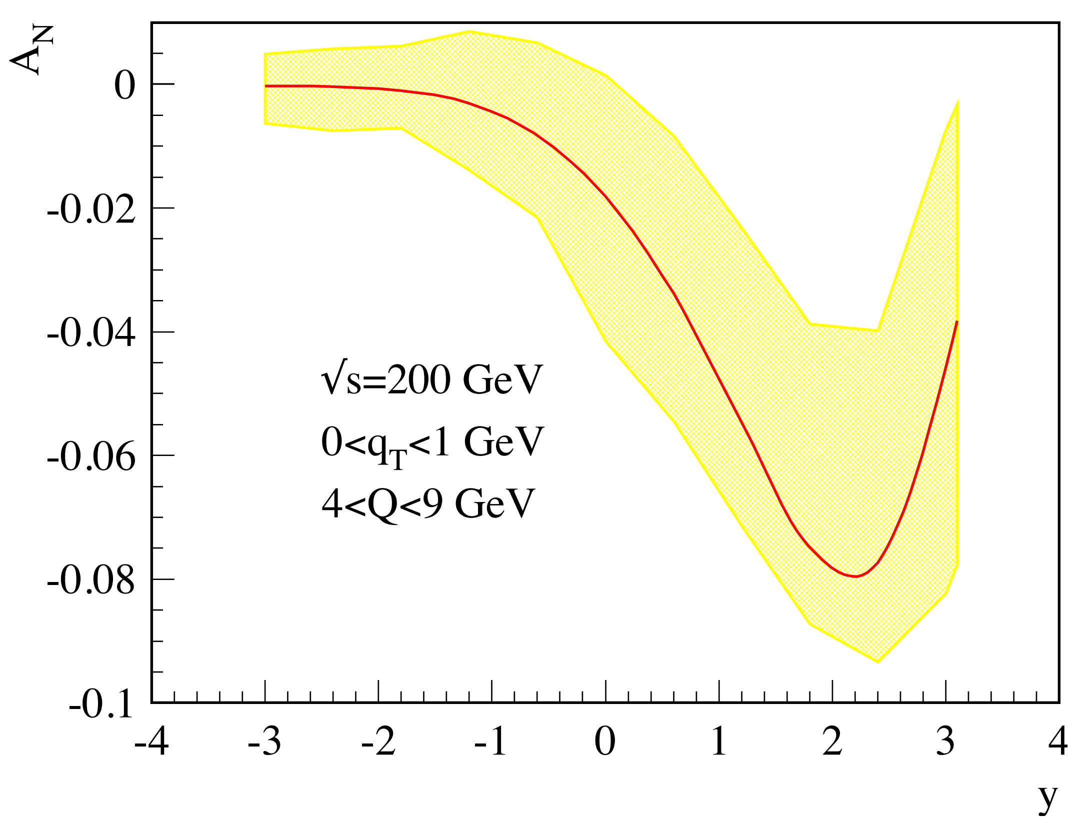
Predictions for Drell-Yan process at RHIC

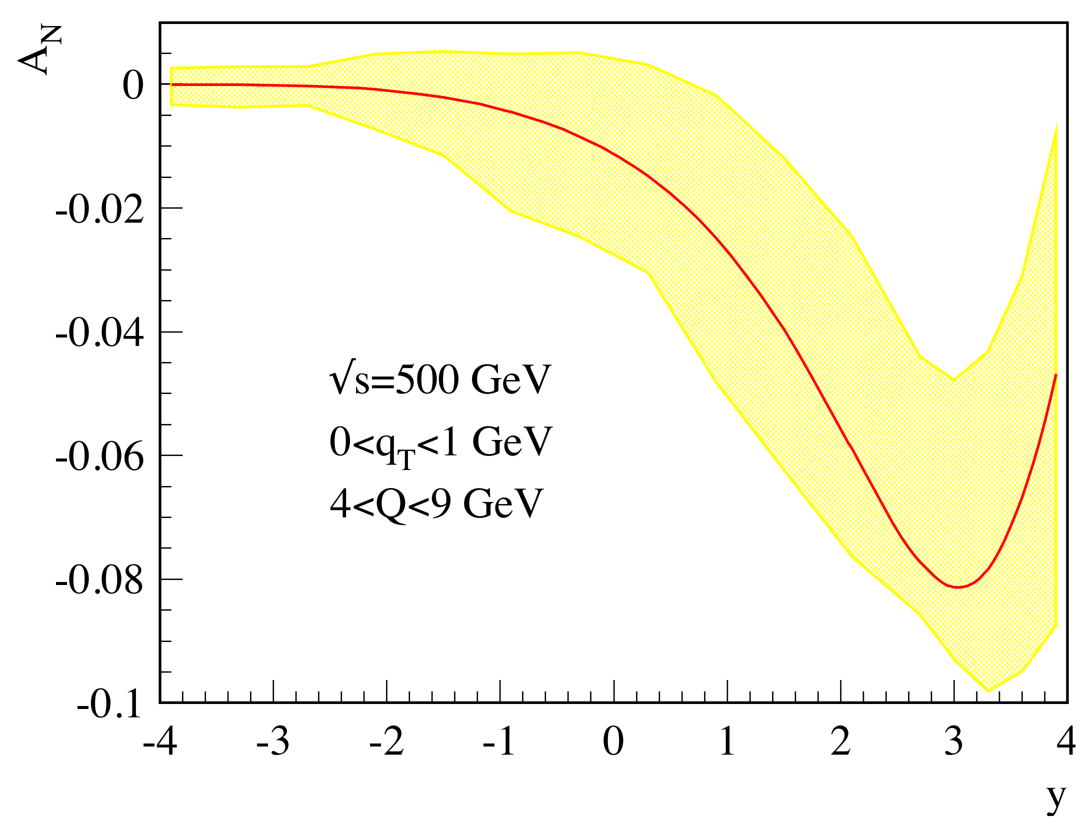
- Reverse the sign of Sivers from SIDIS and make predictions for Drell-Yan production at RHIC

Kang-Qiu, PRD81, 2010



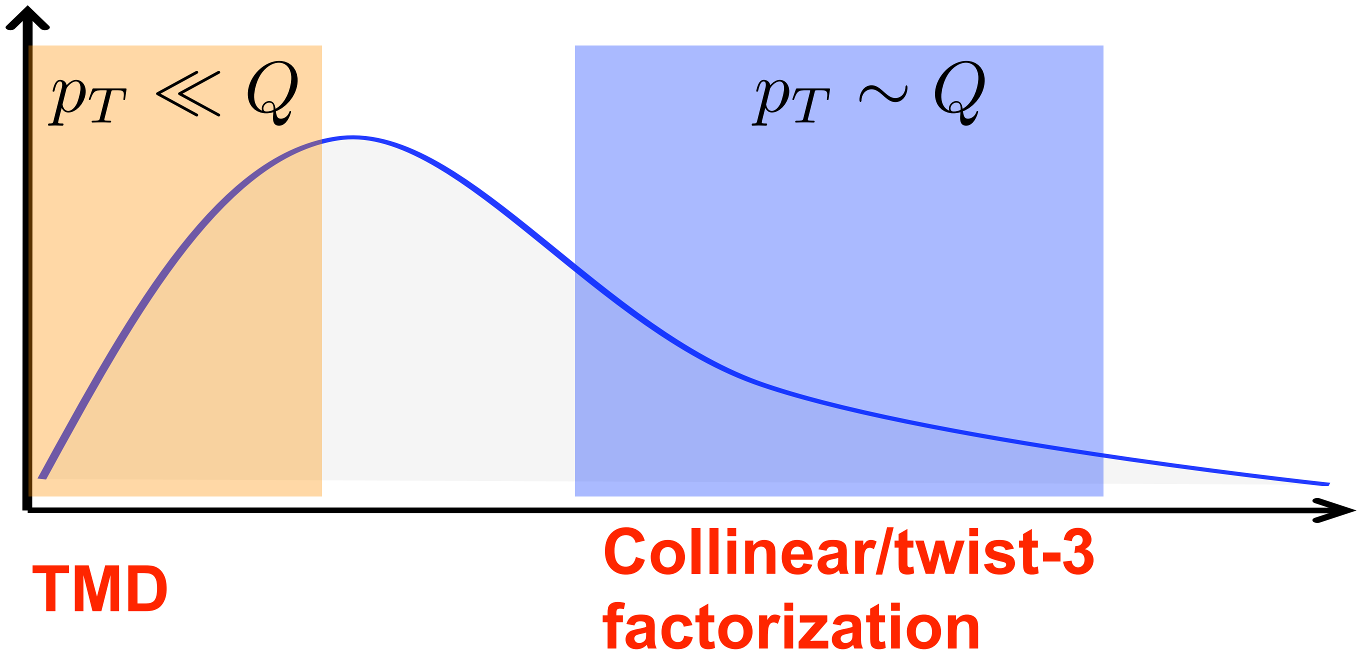
$$A_N \propto \frac{4}{9}\Delta^N u + \frac{1}{9}\Delta^N d < 0$$



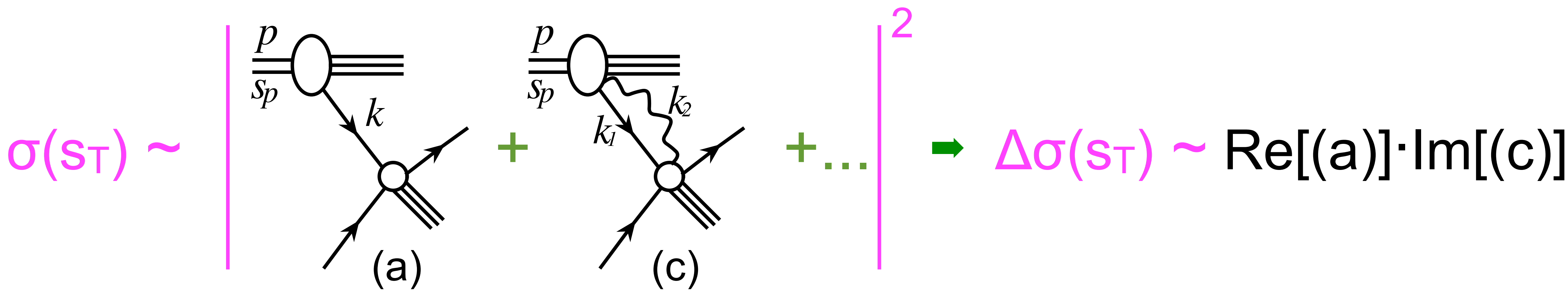


TMD factorization to collinear factorization

- Transition from low p_T to high p_T

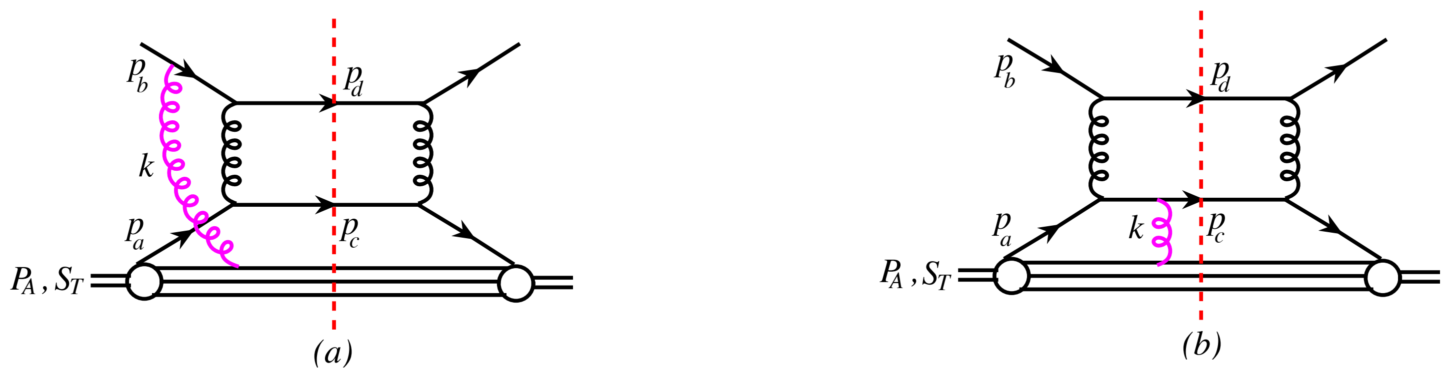


- Collinear twist-3 factorization approach: Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98



Both initial- and final-state interactions

- For the process $pp^\uparrow \to \pi + X$, one of the partonic channel: $qq' \to qq'$



$$E_h \frac{d\Delta\sigma}{d^3P_h} \propto \epsilon^{P_{hT} S_A n \bar{n}} \sum_{a,b,c} D_{h/c}(z_c) \otimes f_{b/B}(x_b) \otimes T_{a,F}(x,x) \otimes H^{\text{Siv}}_{ab\to c}$$

Efremov-Teryaev-Qiu-Sterman (ETQS) function

- The effects of initial- and final-state interaction are absorbed to $H^{\text{Siv}}_{ab\to c}$
- ETQS function $T_{q,F}(x,x)$ is universal

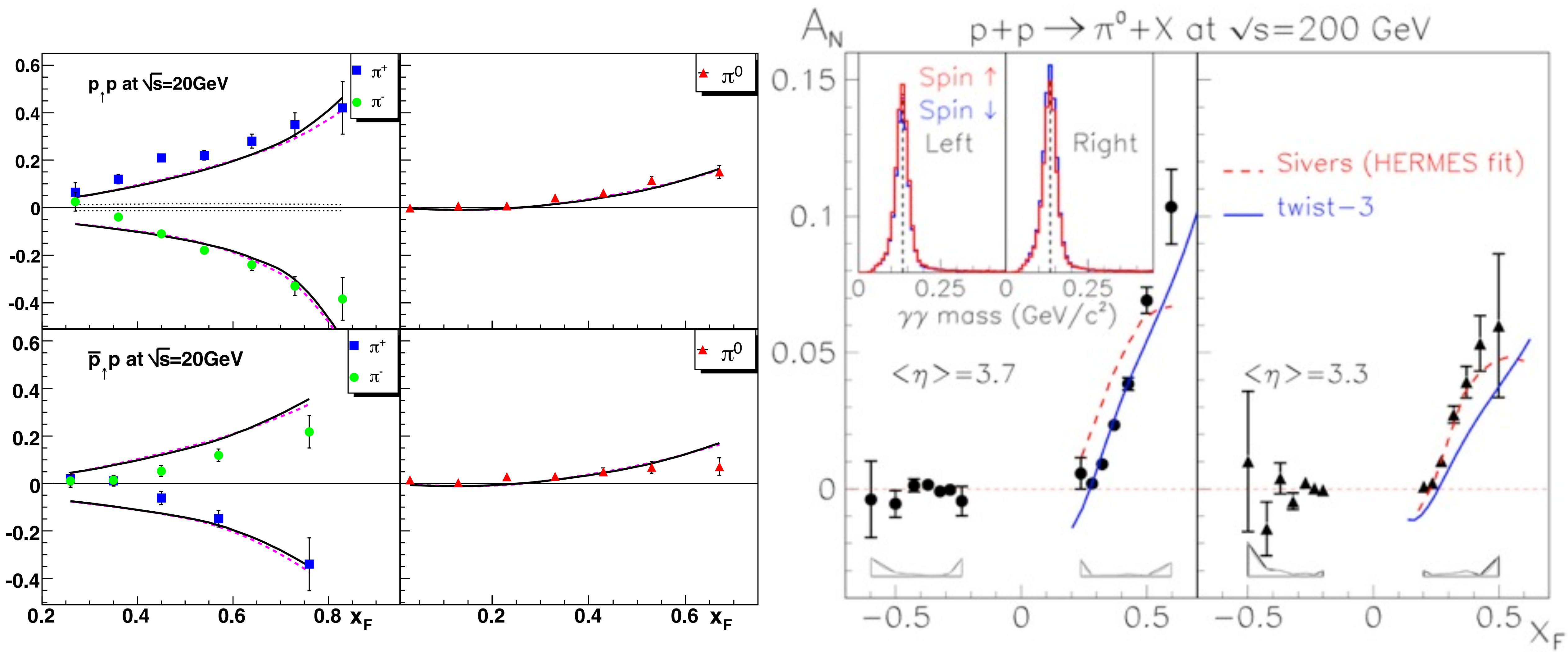
- Since TMD and collinear twist-3 approaches provide a unified picture for the SSAs, ETQS function and Sivers function are closely related to each other

Initial success of twist-3 approach

- Describe both fixed-target and RHIC well: a fit

$$T_{q,F}(x,x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\phi_q(x)$$

Kouvaris-Qiu-Vogelsang-Yuan, 2006

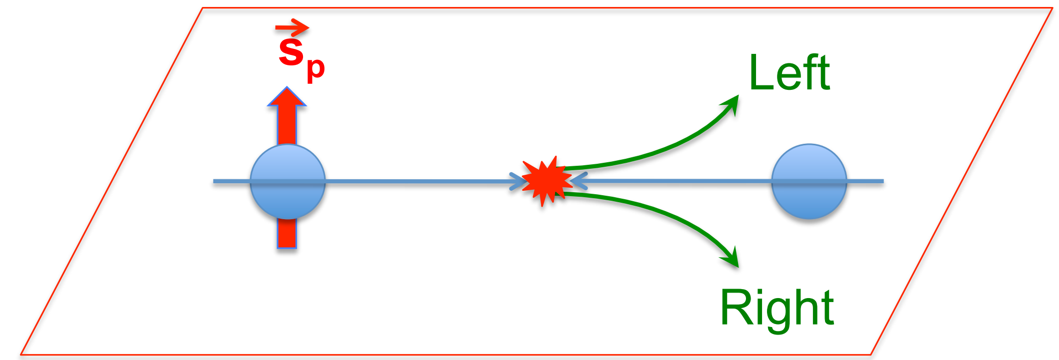


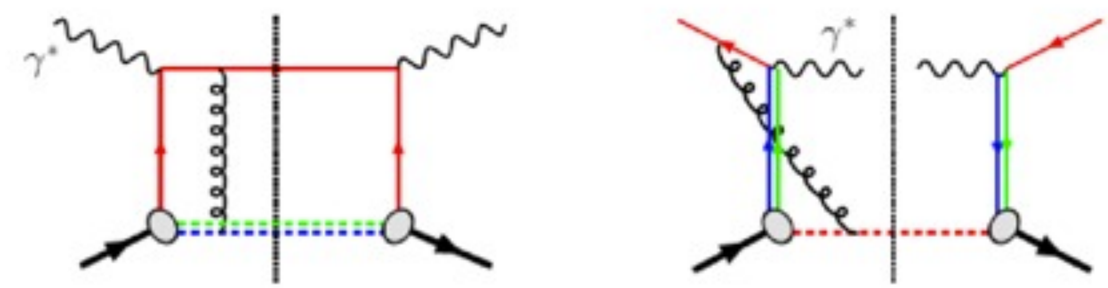
$$p^{\uparrow}p \to \pi + X$$

Sign change and sign mismatch of Sivers effect

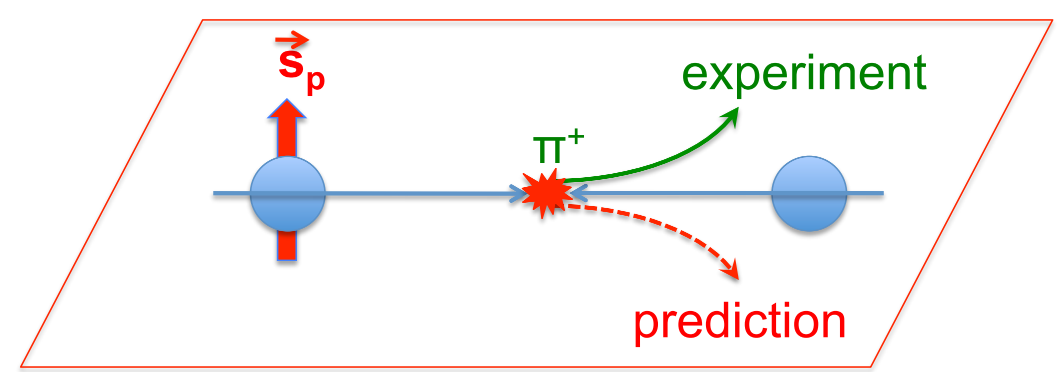
- Single transverse spin asymmetry is a left-right asymmetry
- Sivers effect has been proposed as one of the important contributions
- Sivers function depends on the interaction between the active parton and the remnant
- Final-state interaction in SIDIS and initial-state interaction in DY makes Sivers function opposite
- In pp collision, both FSI and ISI contributes. Take them consistently and use the Sivers function extracted from SIDIS to predict asymmetry in pp, one predicts the particle goes to right while experiments observes them go to left

Kang-Qiu-Vogelsang-Yuan, 2011





$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

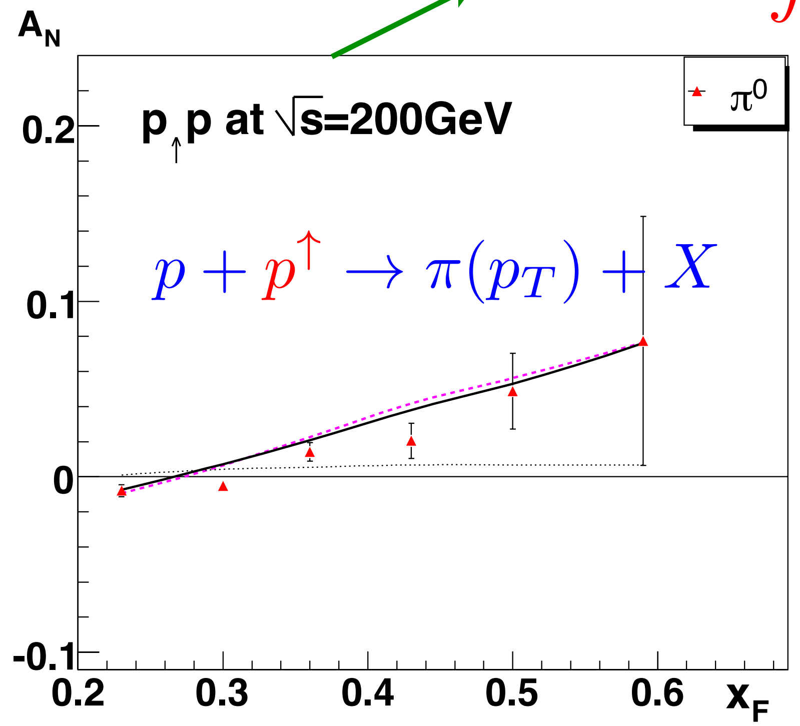


Put in the more formal form

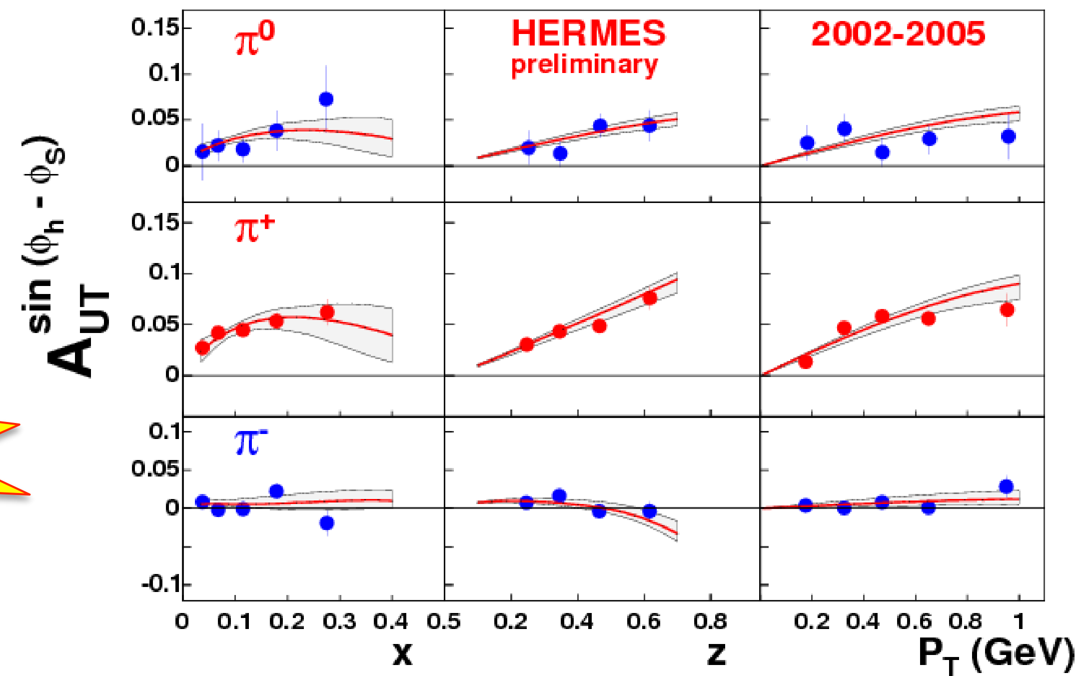
- At the operator level, ETQS function is related to the first kt-moment of the Sivers function

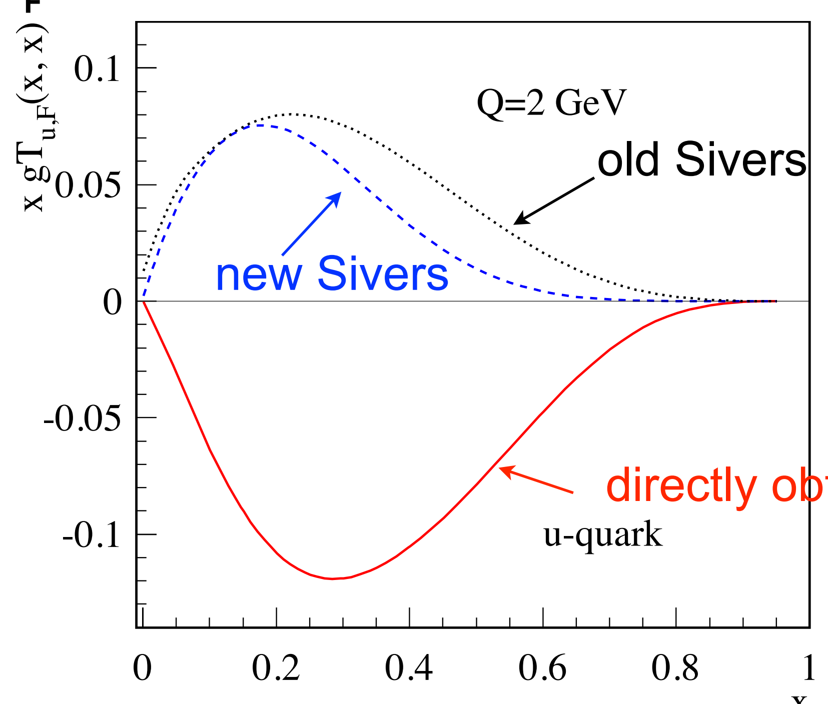
$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}$$

Boer, Mulders, Pijlman, 2003
Ji, Qiu, Vogelsang, Yuan, 2006



Compare

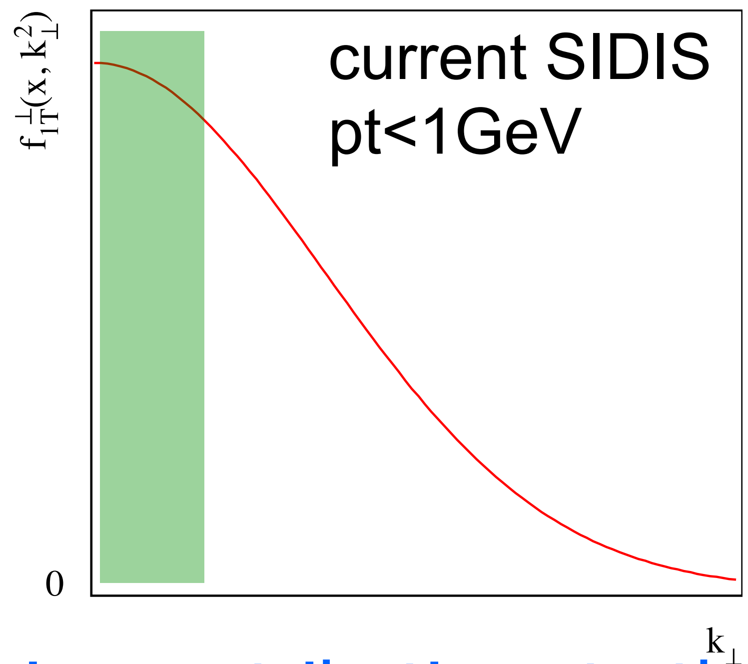


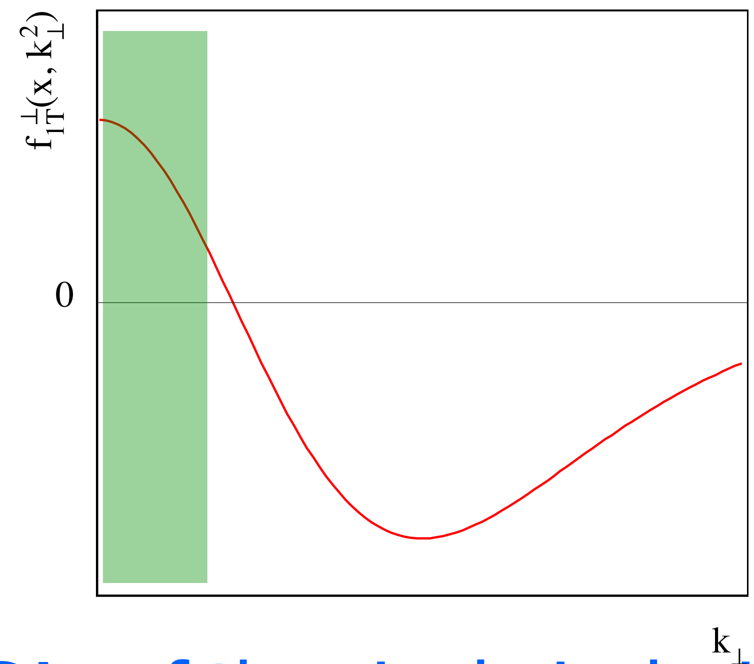


Kang-Qiu-Vogelsang-Yuan, PRD83, 2011

Possible solutions to the sign mismatch

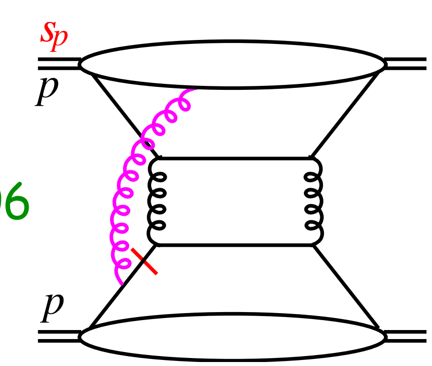
- Different functional form for SIDIS Sivers function
  - node in k_T
  - node in x

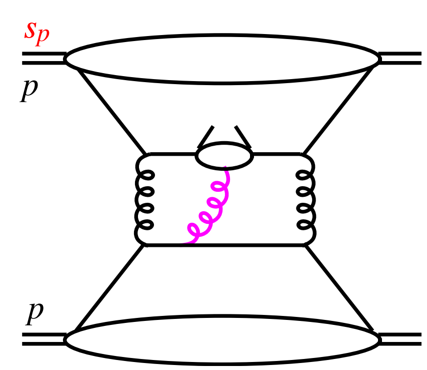




- There are two major contributions to the SSAs of the single inclusive hadron production in pp collisions

Efremov-Teryaev 82, 84,
Qiu-Sterman 91, 98,
Kouvaris-Qiu-Vogelsang-Yuan, 06
Kanazawa-Koike, 11





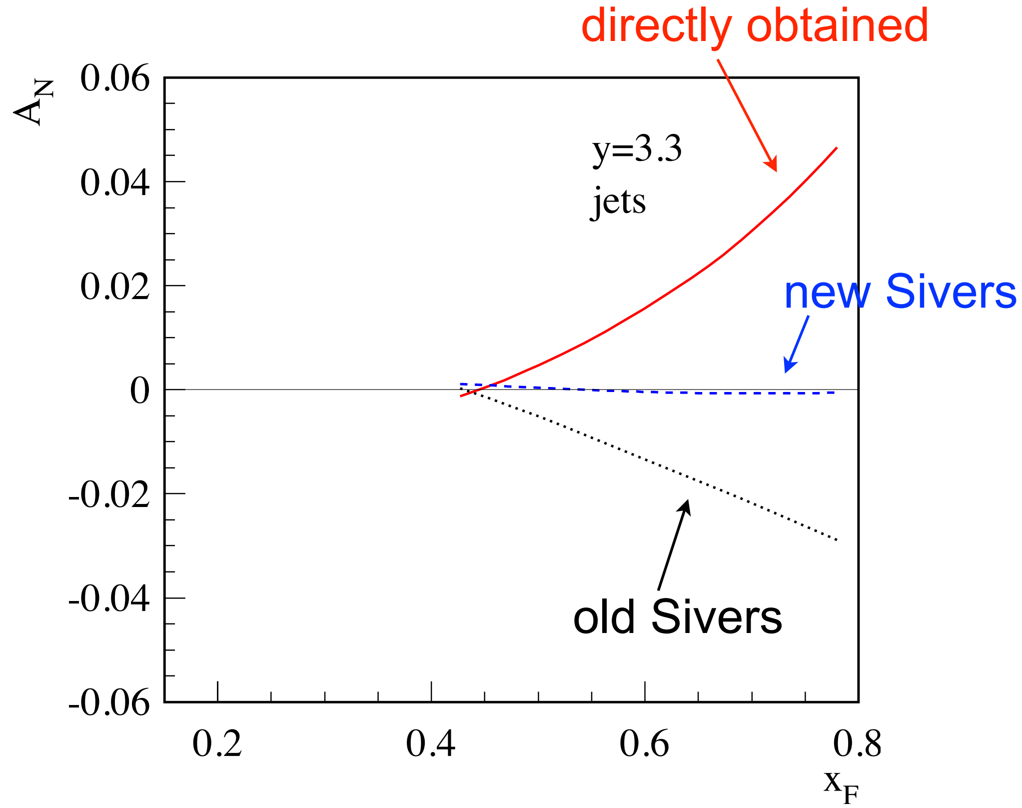
Kang-Yuan-Zhou 2010

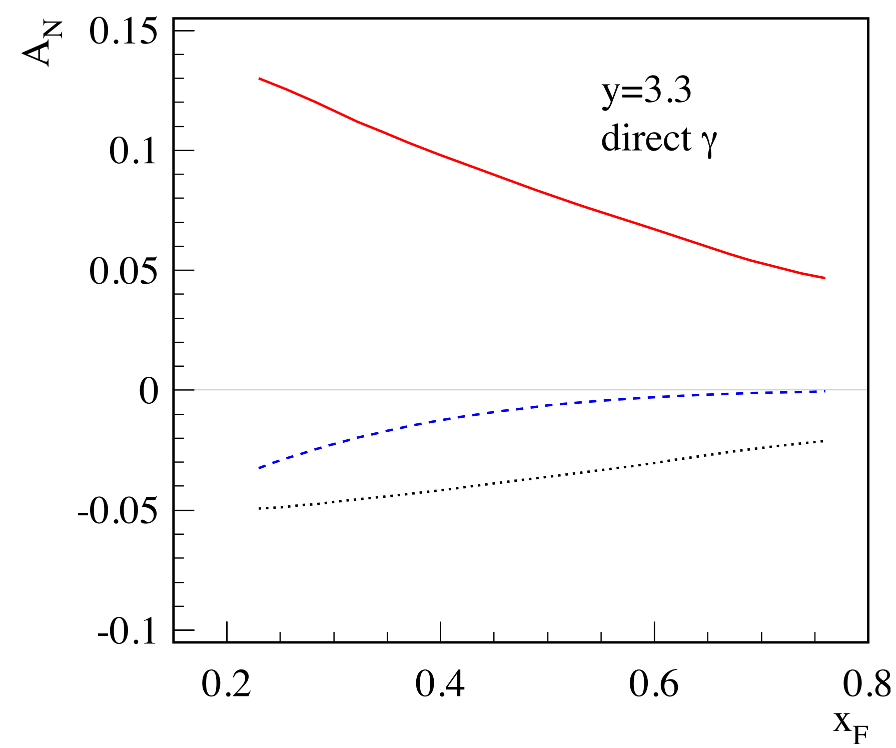
  - The current global fittings are based on the assumptions that the SSAs mainly come from the twist-3 correlation functions, which might not be the case
  - If the contribution from twist-3 fragmentation functions dominates, one might reverse the sign of the ETQS function

Predictions for jet and direct photon

- at RHIC 200 GeV: $p^{\uparrow} + p \to \mathrm{jet} + X$

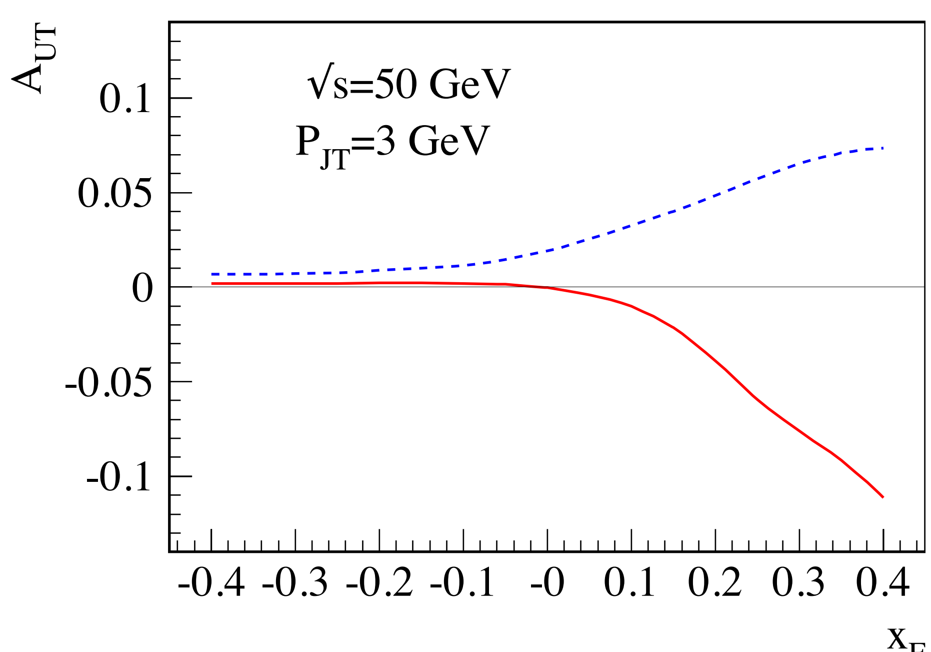
Kang-Qiu-Vogelsang-Yuan, PRD83, 2011

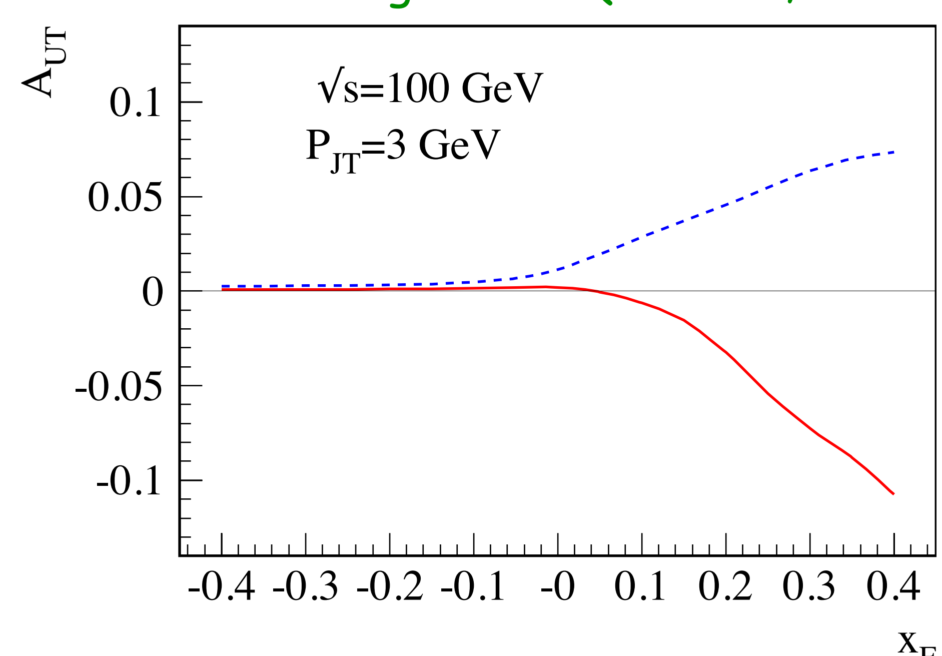




- at EIC: $e + p^{\uparrow} \to \mathrm{jet} + X$ (DIS but with the final lepton undetected)

Kang-Metz-Qiu-Zhou, arXiv: 1106.0266





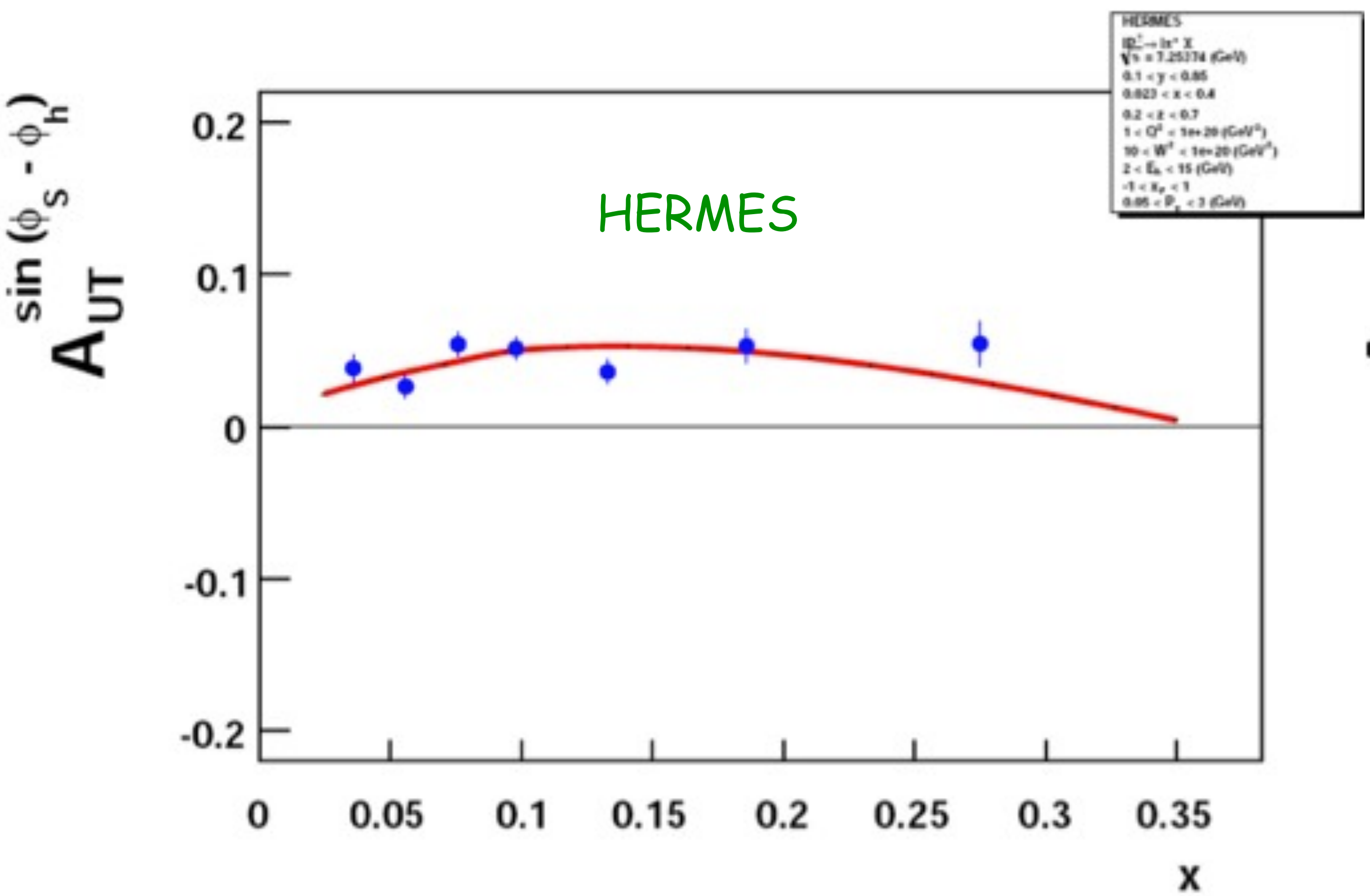
Possible solution: node in x

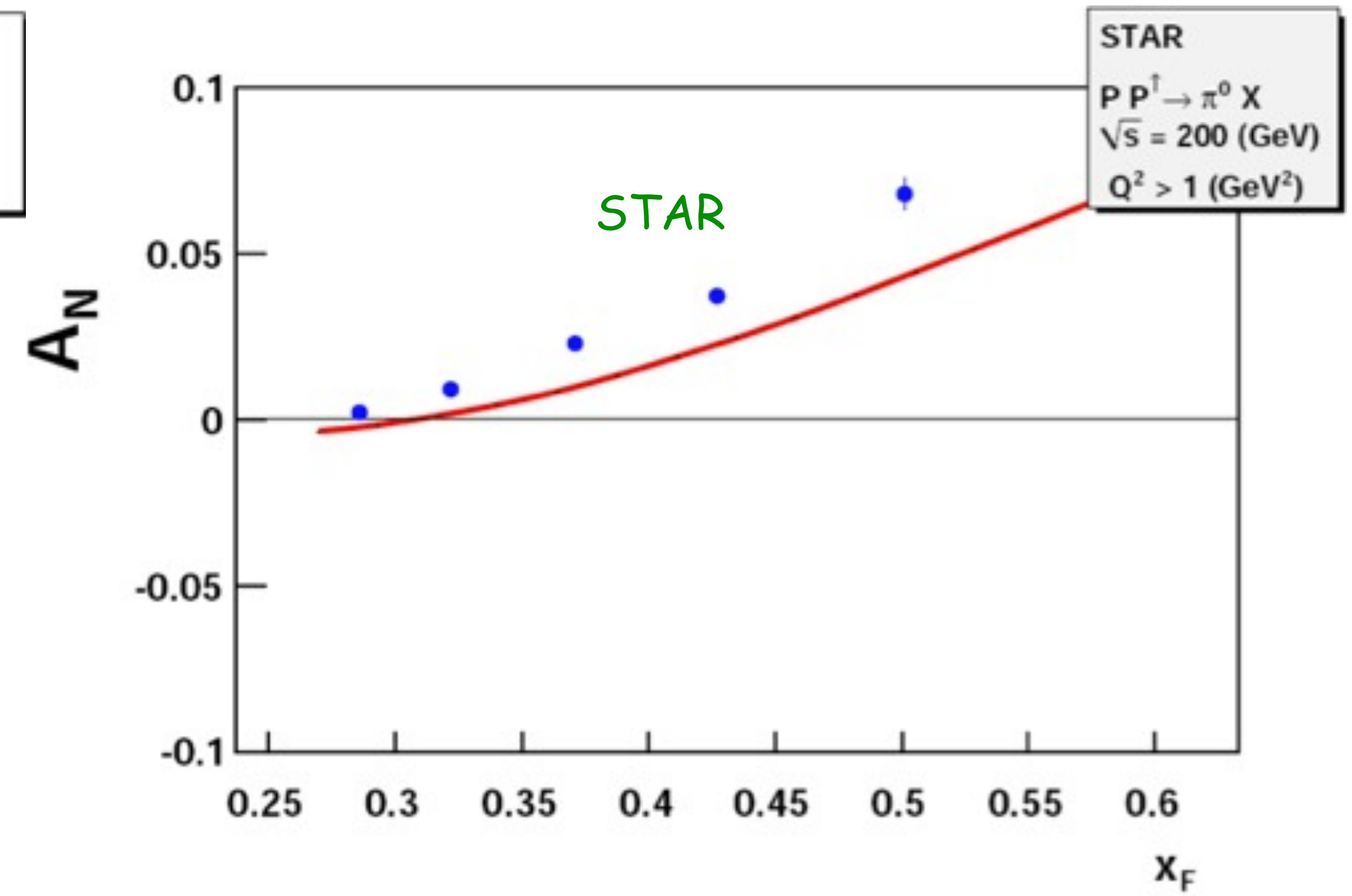
- Use a parametrization which could have a node (similar to DSSV): if η>0, one has a node

Kang-Prokudin, 2011

$$f_{1T}^{\perp q} \propto x^{\alpha_q}(1-x)^{\beta_q}(1-\eta_q x)$$

- Seems describe both HERMES and STAR data reasonably well

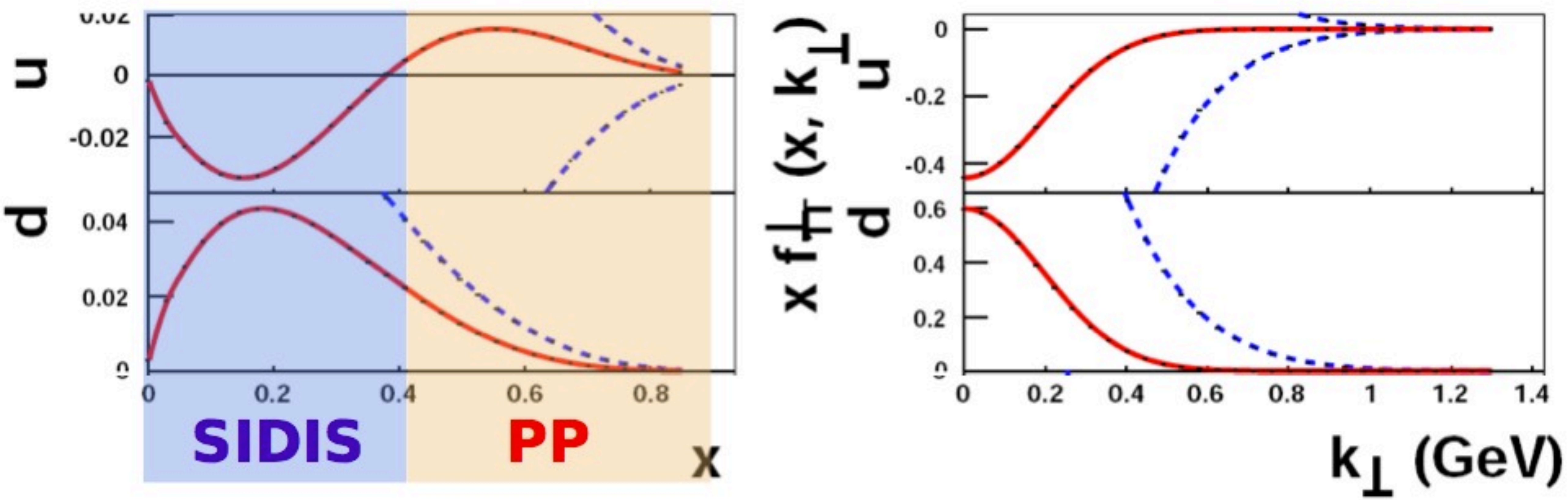




The Sivers function extracted from the fit

- u-quark Sivers function has a node at x~0.4

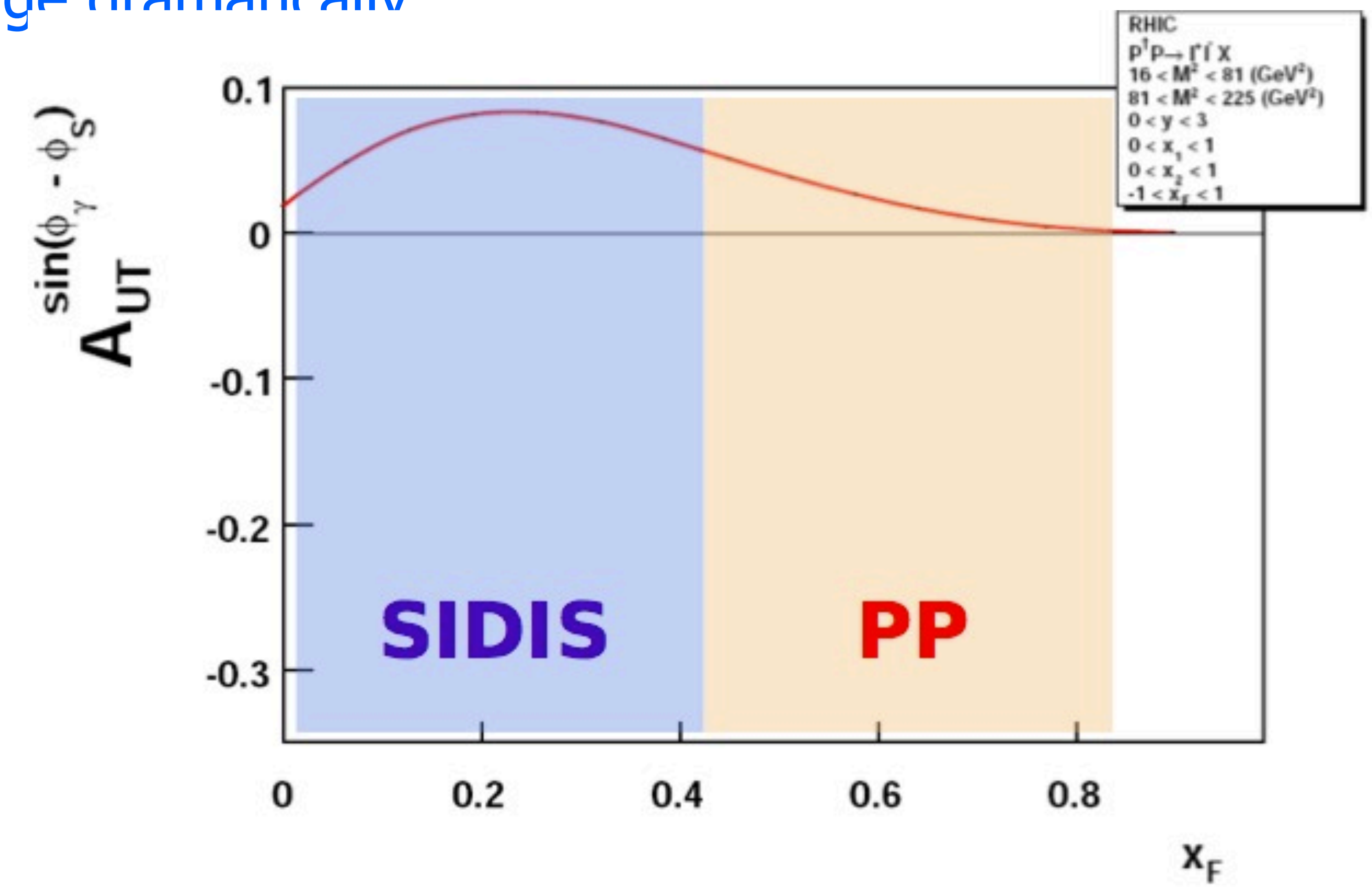
Kang-Prokudin, 2011



The consequence for DY prediction

- With our preliminary fit result, the prediction for the SSA of DY could change dramatically

Kang-Prokudin, 2011

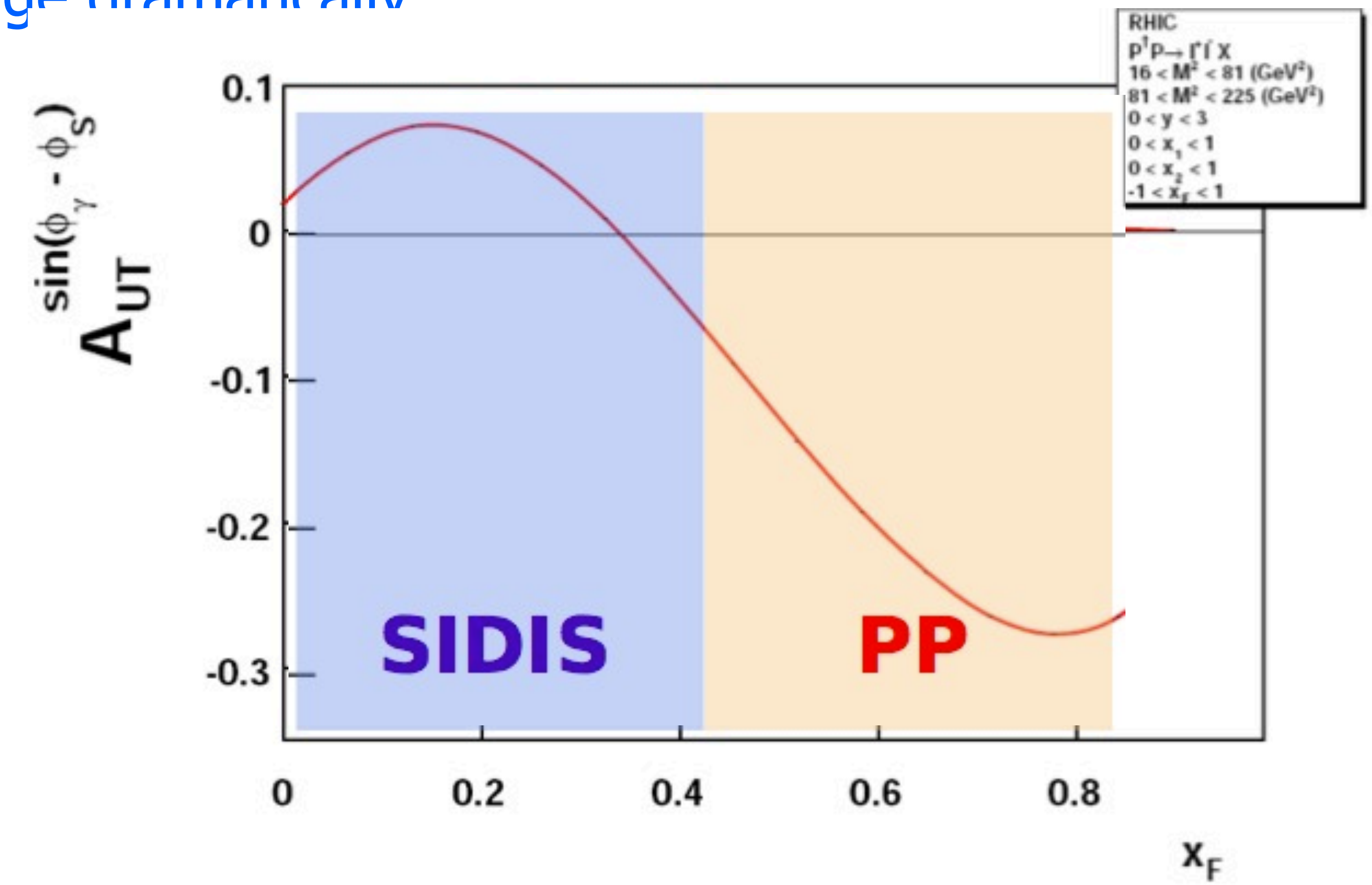


- More studies are needed: in pp collision, there is also twist-3 fragmentation function contribution (analog of the Collins effect), in principle one needs a global fit with SIDIS, pp, and e^+e^- data all included

The consequence for DY prediction

- With our preliminary fit result, the prediction for the SSA of DY could change dramatically

Kang-Prokudin, 2011



- More studies are needed: in pp collision, there is also twist-3 fragmentation function contribution (analog of the Collins effect), in principle one needs a global fit with SIDIS, pp, and e^+e^- data all included

Evolution is very important

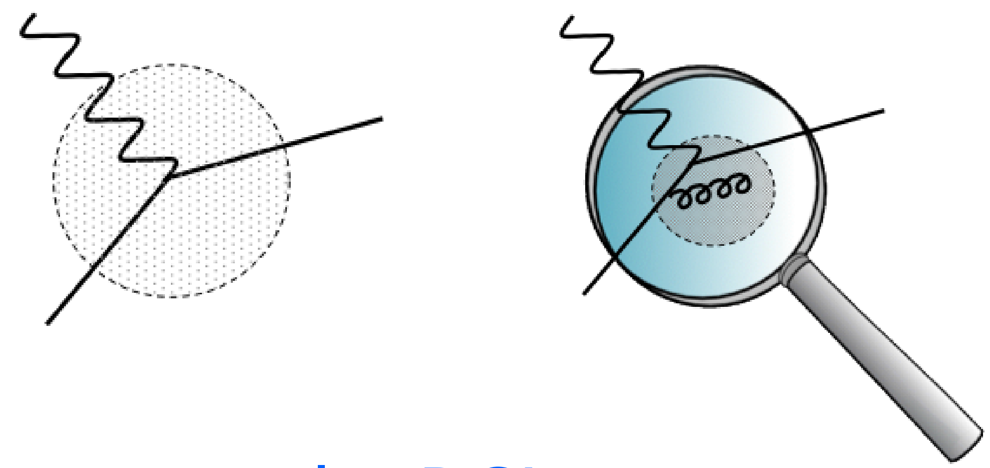
- Most of the predictions are based on lowest-order results (bare parton model), which suffer from strong dependence on factorization scale/ renormalization scale
- To have a reliable prediction, one needs evolution of the correlation functions (for twist-3 approach), and the evolution of the TMDs (for TMD approach)
- Current status - twist-3 side
  - evolution equation for ETQS-function (the first kt-moment of the Sivers function) has been available around 2009

    Kang-Qiu 09, Zhou-Yuan-Liang 09, Vogelsang-Yuan 09, Braun-Manashov-Pirnay 09
  - evolution of twist-3 fragmentation function (the first kt-moment of the Collins function) has been available 2010

    Kang, arXiv: 1012.3419, PRD83, 2011
- Current status - TMD side
  - evolution of unpolarized PDFs: Collins-Soper 81, Aybat-Rogers, arXiv: 1101.5057, …
  - evolution of Sivers function: Kang-Xiao-Yuan, arXiv: 1106.0266

Evolution of collinear functions

- Perturbative change:



- Famous example: DGLAP evolution equation
  - First kt-moment of the Sivers function: similar to unpolarized PDFs

$$\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu)-T_{q,F}(\xi,\xi,\mu)\right]+z\,T_{q,F}(\xi,x,\mu)\right]\right\}$$

  - First kt-moment of the Collins function: similar to transversity

$$\frac{\partial \hat{H}(z_h,\mu)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int\frac{dz}{z}\left\{\delta P_{qq}(\hat{\xi})\hat{H}(z,\mu) + \int\frac{dz_1}{z_1^2}PV\left(\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\right)B(z_h,z,z_1)\hat{H}_F(z,z_1,\mu)\right\}$$

Evolution of TMDs follow Collins-Soper evolution

- Evolution of collinear correlation functions follow the usual DGLAP-type evolution equation, which is equivalent to resum the single-logarithmic contributions to all order

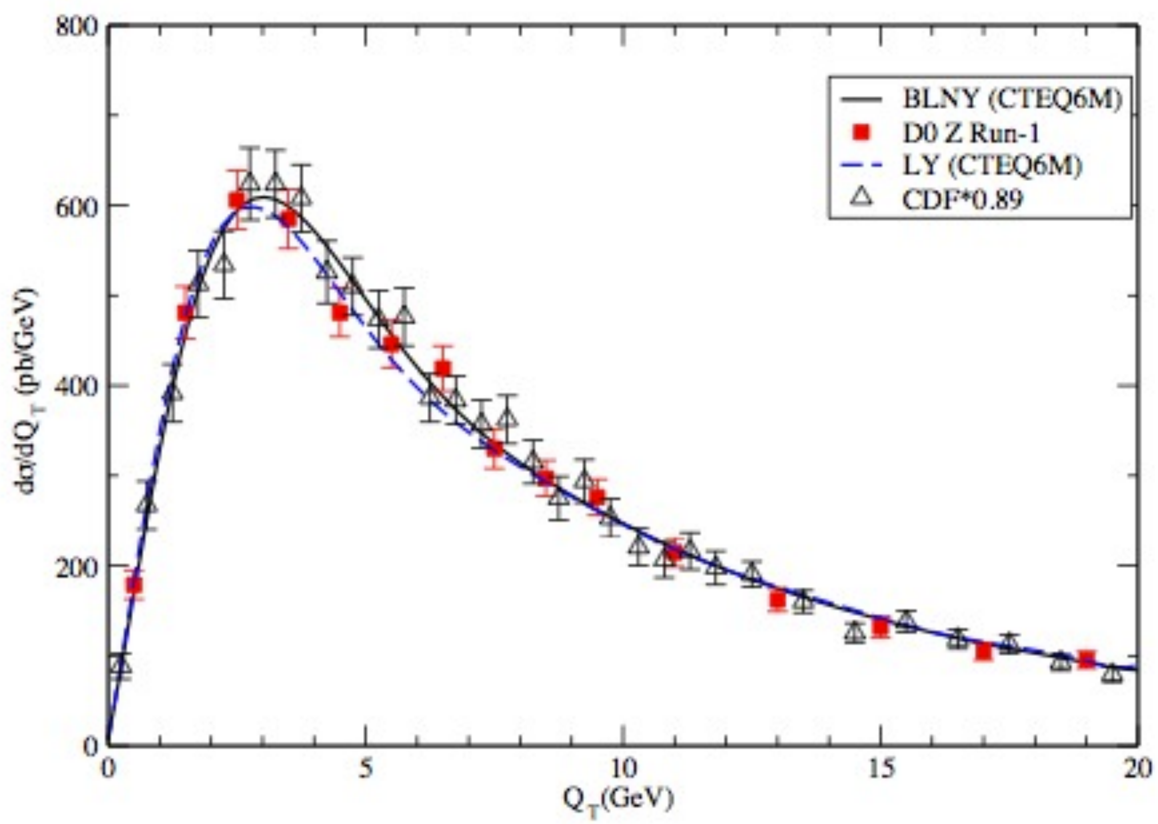
$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

- Evolution of TMDs follow Collins-Soper-type evolution equation, which is equivalent to resum the double-logarithmic contributions to all order, which is usually more difficult

$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

Collins-Soper-Sterman (CSS) formalism

- From the evolution of TMDs, Collins, Soper and Sterman developed a resummation formalism (by solving the Collins-Soper evolution equation for the TMDs) - well-know CSS formalism
  - For example, at RHIC, we use ResBos a lot, which is a numerical implementation of CSS formalism
  - It works perfect for the process with two distinctive scales, like low pt DY, low pt SIDIS, and etc



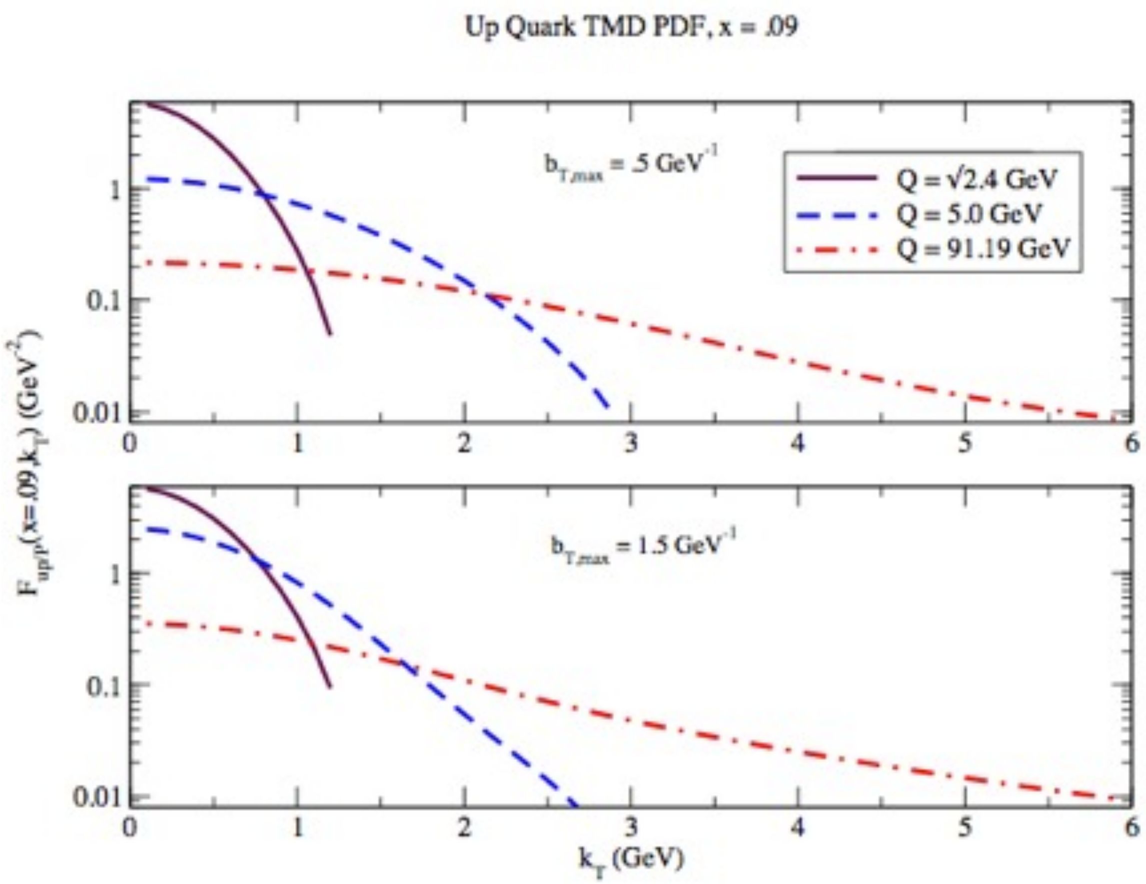
Landry-Brock-Ndaolsky-Yuan, 2002

Evolution of unpolarized TMDs

- In CSS formalism, the evolution of TMDs is taken into account in the intermediate step. Once solving the evolution equation, it becomes the resummation formalism, which depends only on the collinear function
- Aybat and Rogers try to perform this procedure slightly differently. They want to write the final formalism in a TMD form, such that the evolution needs to implement explicitly. However, always remember, no matter what you do, the final result on the observable is equivalent.

Aybat-Rogers, arXiv: 1101.5057

The evolution for unpolarized TMDs



Single spin asymmetry for Drell-Yan production

Kang-Xiao-Yuan, arXiv: 1106.0266

- The TMD formalism:

$$d\Delta\sigma \sim \epsilon^{\beta\alpha} S_\perp^\beta \int d^2\mathbf{k}_{a\perp} d^2\mathbf{k}_{b\perp} d^2\lambda_\perp \delta^2(\mathbf{q}_\perp - \mathbf{k}_{a\perp} - \mathbf{k}_{b\perp} - \lambda_\perp) k_{a\perp}^\alpha f_{1T}^\perp(x_a, k_{a\perp})\, \bar{q}(x_b, k_{b\perp}) S(\lambda_\perp)$$

- In impact-parameter b-space:

$$d\Delta\sigma \sim \epsilon^{\beta\alpha} S_\perp^\beta \tilde{f}_{1T}^{(\perp\alpha)}(z_1, b, \zeta_1) \bar{q}(z_2, b, \zeta_2) H_{UT}(Q)\, (S(b,\rho))$$

- One thus needs to study the evolution of the following factors:
  - Sivers function

$$\tilde{f}_{1T}^{(\perp\alpha)}(x, b, \mu, \zeta) = \frac{1}{M} \int d^2k_\perp e^{-i\vec{k}_\perp \cdot \vec{b}_\perp} k_\perp^\alpha\, f_{1T}^{\perp(\mathrm{DY})}(x, k_\perp, \mu, \zeta)$$

  - Unpolarized quark distribution

$$\bar{q}(z_2, b, \zeta_2)$$

A bit details on the evolution of Sivers function

- Need two steps for the evolution of the quantity which contains double logarithm
  - Sivers function in impact-parameter b-space

$$\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta) = \frac{1}{M}\int d^2k_\perp e^{-i\vec{k}_\perp\cdot\vec{b}_\perp} k_\perp^\alpha\, f_{1T}^{\perp(\mathrm{DY})}(x,k_\perp,\mu,\zeta)$$

  - It follows Collins-Soper evolution equation

$$\zeta\frac{\partial}{\partial\zeta}\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta) = [K(\mu,b)+G(\mu,\zeta)]\,\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)$$

  - Since it contains double-logarithm, the evolution kernel also contains "single-logarithm", which follows a further evolution equation

$$\mu\frac{d}{d\mu}K(\mu,b) = -\gamma_K = -\mu\frac{d}{d\mu}G(\mu,\zeta) \qquad\qquad \gamma_K = \frac{\alpha_s}{\pi}2C_F$$

  - Now one could solve this evolution equation to resum the double logarithms

$$\begin{aligned}\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,x\zeta) &= \exp\left\{-\int_{\mu_L}^{C_2 x\zeta}\frac{d\mu}{\mu}\left[\ln\left(\frac{C_2 x\zeta}{\mu}\right)\gamma_K(\alpha_s(\mu)) - K(\mu_L,b) - G(\mu,\mu/C_2)\right]\right\}\\ &\times\ \tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,x\zeta_0=\mu_L/C_2)\end{aligned}$$

Take the evolution of all TMDs

- The final result is a resummation formalism for the SSA of DY
  - Sudakov resummation

$$\widetilde{W}^{\alpha}_{UT}(Q;b) = e^{-\mathcal{S}_{UT}(Q^2,b)}\widetilde{W}^{\alpha}_{UT}(C_1/b,b)$$
$$= (-ib^{\alpha}_{\perp}/2)\, e^{-\mathcal{S}_{UT}(Q^2,b)}\Sigma_{i,j}\Delta C^{T}_{qi}\otimes T_{i,F}(z_1,z_1)C_{\bar{q}j}\otimes f_{j/B}(z_2)$$

  - Sudakov exponent: all the double logarithms are resummed to all orders

$$\mathcal{S}_{UT}(Q^2,b) = \int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{C_2^2Q^2}{\mu^2}\right)A_{UT}(C_1;g(\mu)) + B_{UT}(C_1,C_2;g(\mu))\right]$$

$$A = \sum_{n=1} A^{(n)}(\alpha_s/\pi)^n$$

$$A^{(1)}_{UT} = C_F \qquad B^{(1)}_{UT} = -\frac{3}{2}C_F$$

$$\Delta C^{T(0)}_{qq} = \delta(1-x) \qquad \Delta C^{T(1)}_{qq} = -\frac{1}{4N_c}(1-x) + \frac{C_F}{2}\delta(x-1)\left[\frac{\pi^2}{2}-4\right]$$

- This is the spin-analog of CSS formalism

Features of this formalism

- Once perform Fourier transform from b-space back to qt-space, one could predict the qt-dependence of the SSAs of DY (underway)

$$\frac{d\Delta\sigma(S_\perp)}{dy dQ^2 d^2 q_\perp} = \sigma_0 \epsilon^{\alpha\beta} S_\perp^\alpha W_{UT}^\beta(Q; q_\perp) \ ,$$

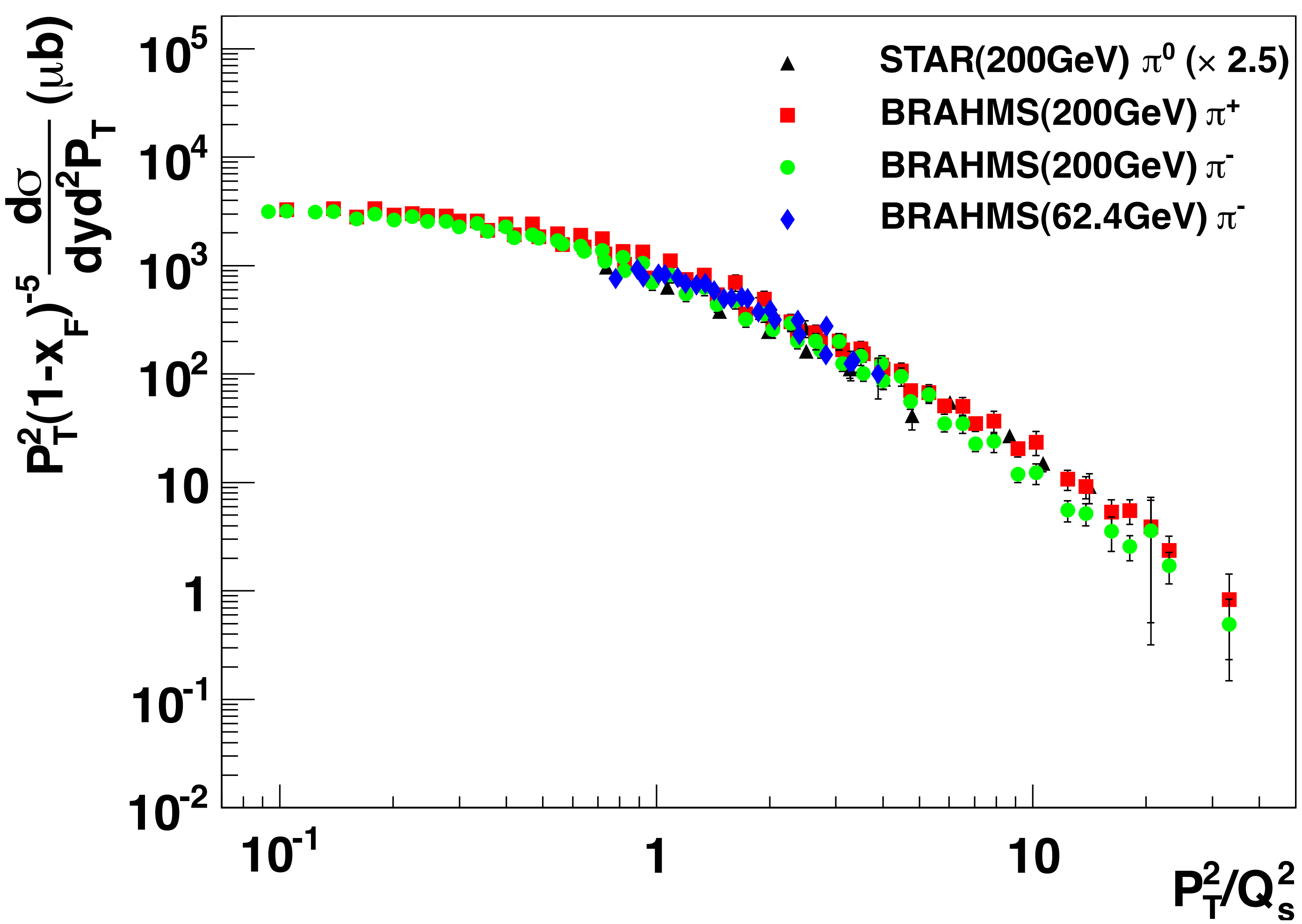
$$W_{UT}^\alpha(Q; q_\perp) = \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{b}} \widetilde{W}_{UT}^\alpha(Q; b) + Y_{UT}^\alpha(Q; q_\perp)$$

- The resummation formalism, just like the conventional CSS formalism, depends only on the collinear correlation function
- Most importantly, this formalism depends on ETQS function $T_F(x, x)$, which is universal
- The sign change from SIDIS to DY is contained in the short-distance part (coefficient function)
- It becomes very important to determine the sign of ETQS function, to better predict the SSAs of DY (thus to test the sign change)

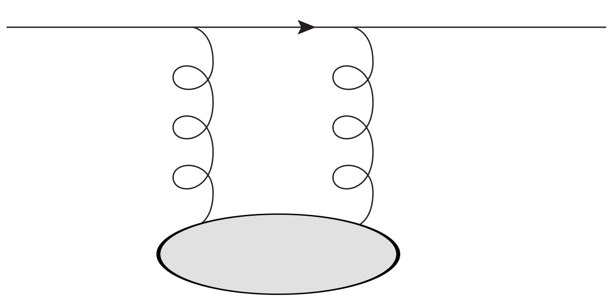
Connection of spin asymmetry to small-x physics

- Since the single transverse spin asymmetry becomes large at very forward region, which probes the small-x region of the parton in the target nucleon
- Could they have connection?
- Unpolarized cross section at forward seems follow a geometric scaling

$$\frac{d\sigma}{dy_h d^2 P_{h\perp}} = \frac{K}{(2\pi)^2} \int_{x_F}^{1} \frac{dz}{z^2} \int d^2 P_{hT} x_1 q(x_1) N_F(x_2, k_\perp) D_{h/q}(z, P_{hT})$$



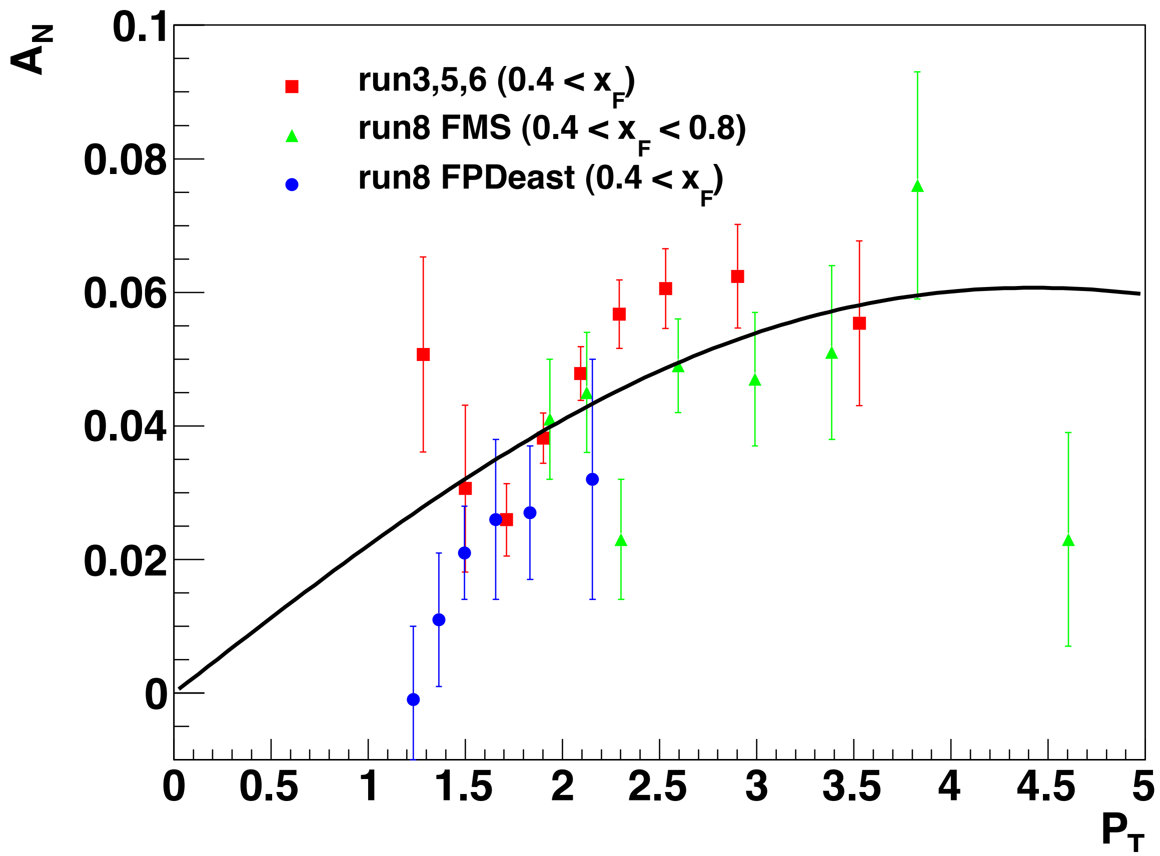
Kang-Yuan, arXiv: 1106.1375

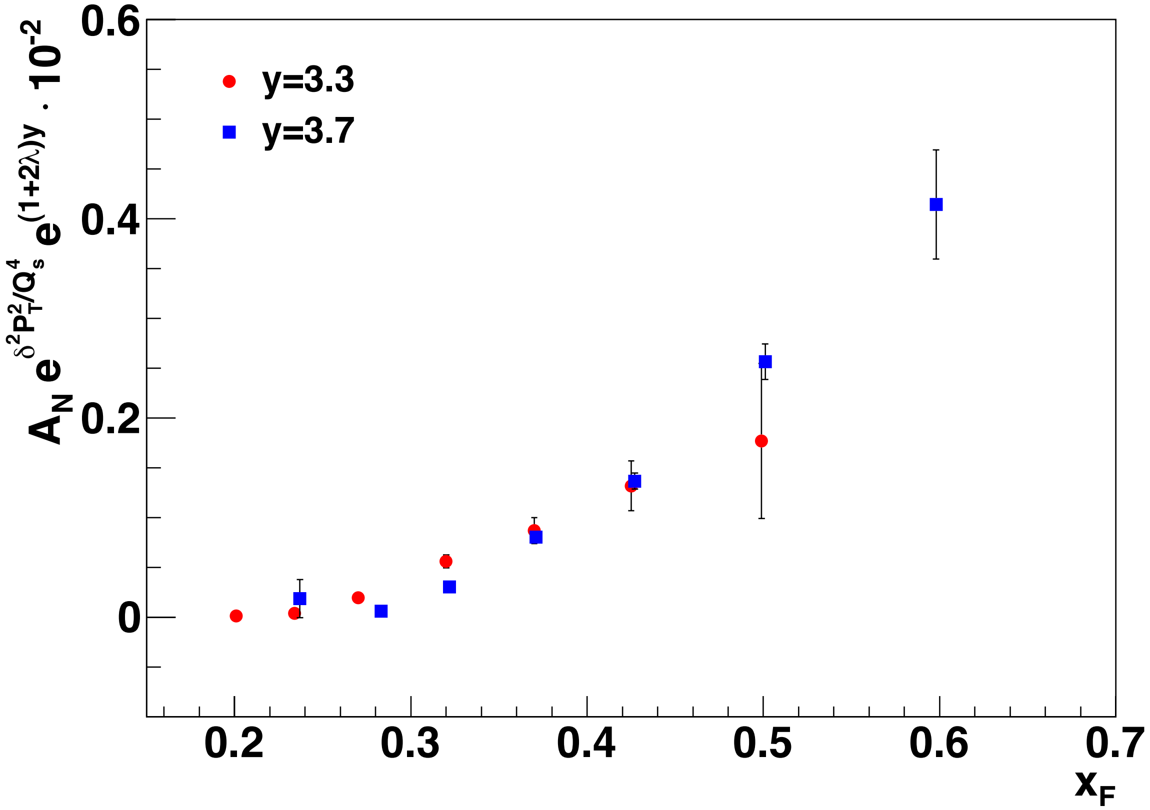


Data seems support scaling analysis

- Scaling analysis for pt- and xf-dependence

$$\frac{d\Delta\sigma}{dy_h d^2P_{h\perp}} = \frac{K}{(2\pi)^2}\int_{x_F}^{1}\frac{dz}{z^2}\int d^2P_{hT} I(S_\perp, P_{hT}) x_1 h(x_1) N_F(x_2, k_\perp)\delta\hat{q}(z, P_{hT})$$





$$A_N \sim \frac{P_{h\perp}\Delta}{Q_s^2} e^{-\frac{\delta^2 P_{h\perp}^2}{(Q_s^2)^2}}$$

$$A_N e^{\delta^2 P_{h\perp}^2/Q_s^4} e^{(1+2\lambda)y_h} \sim x_F^{(1+\lambda)}\mathcal{F}(x_F)$$

Summary

- The existence of Sivers function relies on the initial and final-state interactions
- Sivers effect is process dependent
  - Test process-dependence is very important to understand the SSAs: sign change between SIDIS and DY
  - Both TMD and collinear twist-3 approaches seem to be successful phenomenologically
- Their connection seems to have a puzzle
- Evolution of the relevant functions has now become available
- SSAs might be closely related to small-x physics
- Hope more and better experimental data coming soon in the near future

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Thank you